



Universidad Autónoma de San Luis Potosí
Facultad de Ingeniería
Centro de Investigación y Estudios de Posgrado

**Metodología para la implementación de láminas con memoria
de forma como medio de accionamiento de una pinza flexible.**

**Methodology for the implementation of shape memory alloy
sheets as the activation method of a compliant gripper.**

T E S I S

Que para obtener el grado de:

Maestría en Ingeniería Mecánica

Presenta:

Ing. Martín Enrique Morales Montelongo

Asesor:

Dr. Gilberto Mejía Rodríguez

San Luis Potosí, S. L. P.

Febrero de 2025



21 de marzo de 2024

ING. MARTÍN ENRIQUE MORALES MONTELONGO
P R E S E N T E.

En atención a su solicitud de Temario, presentada por el **Dr. Gilberto Mejía Rodríguez**, Asesor de la Tesis que desarrollará Usted, con el objeto de obtener el Grado de **Maestro en Ingeniería Mecánica**. Me es grato comunicarle que en la sesión del H. Consejo Técnico Consultivo celebrada el día 21 de marzo del presente, fue aprobado el Temario propuesto:

TEMARIO:

"Metodología para la implementación de láminas con memoria de forma como medio de accionamiento de una pinza flexible"

1. Introducción.
2. Antecedentes teóricos.
3. El modelo de cuerpo pseudo – rígido como una herramienta en el análisis de largas deformaciones de vigas.
4. Simulación numérica de una lámina en voladizo de aleación con memoria de forma basada en datos experimentales.
5. Implementación de técnicas de optimización para el diseño de una pinza flexible basada en aleaciones con memoria de forma.
6. Metodología propuesta para el diseño de una pinza flexible basada en aleaciones con memoria de forma.
7. Conclusiones.
8. Trabajo a futuro.
9. Referencias.

"MODOS ET CUNCTARUM RERUM MENSURAS AUDEBO"

ATENTAMENTE


DR. EMILIO JORGE GONZÁLEZ GALVÁN
DIRECTOR



www.uaslp.mx

Av. Manuel Nava 8
Zona Universitaria • CP 78290
San Luis Potosí, S.L.P.
tel. (444) 826 2330 al 39
fax (444) 826 2336

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FACULTAD DE INGENIERÍA
Área de Investigación y Estudios de Posgrado

D E C L A R A C I Ó N

El presente trabajo que lleva por título:

Metodología para la implementación de láminas con memoria de forma como medio de accionamiento de una pinza flexible.

se realizó en el periodo enero de 2022 a febrero de 2025 bajo la dirección del Dr. Gilberto Mejía Rodríguez.

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RESUMEN.

Los mecanismos flexibles basados en aleaciones con memoria de forma son una interesante alternativa a los mecanismos de cuerpo rígido debido al fenómeno de efecto de memoria de forma. El efecto de memoria de forma se relaciona a la habilidad de una aleación con memoria de forma para recuperar su forma original a través de un cambio de temperatura, de una temperatura baja en el estado de martensita a una temperatura elevada en el estado de austenita, siendo capaz incluso de proveer una fuerza en la dirección de recuperación. Este proyecto de tesis está enfocado en especificar los pasos requeridos para implementar aleaciones con memoria de forma como el método de activación de una pinza flexible utilizando modelos matemáticos y simulaciones numéricas. El modelo de cuerpo pseudo-rígido se utiliza para calcular largas deformaciones de actuadores de aleaciones con memoria de forma de Ni-Ti, que permiten el uso del efecto de memoria de forma para proveer la fuerza de sujeción de la pinza flexible. Se utiliza el análisis de simulaciones numéricas para estimar las propiedades mecánicas de los materiales de aleación con memoria de forma a partir de resultados experimentales para poder predecir la fuerza de salida que los actuadores son capaces de entregar. A su vez, estos tipos de análisis son útiles para estimar el desempeño del prototipo del sujetador antes de su construcción. Algoritmos de optimización son utilizados para calcular las condiciones requeridas de carga necesarias para los actuadores para entregar una fuerza de salida objetivo previamente especificada. La metodología propuesta para la implementación de láminas de aleación con memoria de forma como método de activación de un sujetador flexible se prueba con dos casos de estudio demostrando resultados satisfactorios que a la vez proveen una guía para trabajo a futuro.

ABSTRACT.

Shape memory alloy (SMA)-based compliant mechanisms (CMs) are an interesting alternative to rigid body mechanisms due to the shape memory effect (SME) phenomenon. The SME is related to the ability of a SMA to restore its original shape through a change of temperature from a low temperature in the martensitic state to a high temperature in the austenitic state, while being capable of delivering a force in the recovery direction. This thesis project aims to specify the required steps to implement SMAs as the activation method of a compliant gripper using mathematical models and numerical simulations. The pseudo-rigid body model (PRBM) is used to calculate the large displacements of Ni-Ti SMA actuators, which allow the use of the SME to deliver the grasping force of the compliant gripper. Numerical simulation analyses are used to estimate the mechanical properties of the SMA materials from experimental data to be able to predict the output force that the actuators are capable of delivering. At the same time, these types of analyses are useful to estimate the performance of the gripper prototype before building it. Optimization algorithms are used to calculate the required loading conditions for the actuators to deliver a previously specified objective output force. The proposed methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper is tested with two case studies showing satisfactory results and providing guidance for future work.

ACKNOWLEDGEMENTS.

Thanks to my parents Martín Morales Delgado and María Sandra Montelongo Estrada for their unconditional support during my studies, for pushing me to continue working and accomplish my goals and for teaching me to never surrender and finish all of my objectives, apart from giving me their unconditional love and trust which inspires me to make them proud.

To my sister Sandra Judith Morales Montelongo for giving me her support and company and for inspiring me to continue my professional education.

To my girlfriend Donna Michelle Cortés Méndez for all the love and company that she gives me constantly, I appreciate her patience, comprehension and unconditional support which helped me to always continue forwards regardless of how difficult it is.

To my advisor Dr. Gilberto Mejía Rodríguez for his support, patience and great dedication, I am also grateful for his guidance which helped me to continue my work and to find a way of solving all of the obstacles related to this thesis project.

To the CONACyT for giving me financial support during my studies.

To the Academic Committee of the Mechanical Engineering Postgraduate for their contributions to my academic formation.

To the Faculty of Engineering for providing me with the necessary means to grow professionally.

Martín Enrique Morales Montelongo.

AGRADECIMIENTOS.

Agradezco a mis padres Martín Morales Delgado y María Sandra Montelongo Estrada por su apoyo incondicional durante mis estudios, por impulsarme a seguir trabajando en cumplir mis metas y enseñarme a jamás rendirme hasta terminar todos mis objetivos, además de entregarme su amor incondicional y confianza lo cual me impulsa a hacerlos sentir orgullosos.

A mi hermana Sandra Judith Morales Montelongo por siempre brindarme su apoyo y su compañía e inspirarme a seguir desarrollándome de manera profesional.

A mi novia Donna Michelle Cortés Méndez por todo el amor y compañía que me brinda constantemente, agradezco su paciencia, comprensión y apoyo incondicional que me ayudó a siempre seguir adelante a pesar de las dificultades.

A mi asesor Dr. Gilberto Mejía Rodríguez por su apoyo, paciencia y gran dedicación, también agradezco su guía con la que siempre pude seguir adelante y encontrar la manera de superar todos los obstáculos relacionados a este proyecto de tesis.

Al Consejo Nacional de Ciencia y Tecnología (CONACyT) por brindarme apoyo económico durante mis estudios.

Al Comité Académico del Posgrado en Ingeniería Mecánica por su contribución a mi formación académica.

A la Facultad de Ingeniería por brindarme los medios necesarios para desarrollarme profesionalmente.

Martín Enrique Morales Montelongo.

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NOMENCLATURE.

Symbol	Meaning
%e	Absolute percentage error
A_{eq}	Equality constraints coefficients matrix
A_f	Austenite finish temperature
A_{ineq}	Inequality constraints coefficient matrix
A_s	Austenite start temperature
b	Cross section base
b_{eq}	Equality constraints results vector
b_{ineq}	Inequality constraints results vector
DX	Horizontal beam deflection variable
DY	Vertical beam deflection variable
E	Elastic modulus
E_a	Austenite phase elastic modulus
E_m	Martensite phase elastic modulus
E_t	Tangent modulus
F	Force load
F_{in}	Input force
F_{out}	Output force
F_s	Spring force
fun	Objective function of the optimization problem
h	Cross section height
I	Beam cross section moment of inertia
k	Force-displacement curve elastic slope
k_a	Austenite phase force-displacement curve elastic slope
k_m	Martensite phase force-displacement curve elastic slope
k_s	Spring constant
l	Beam length
lb	Lower bound for each problem variable
M	Moment load
M_f	Martensite finish temperature
M_p	Plastic moment
M_s	Martensite start temperature
MTS	Maximum transformation strain
nonlcon	Non-linear constraints of the optimization problem
nvars	Number of optimization variables
P_y	Vertical position
R_{fs}	Fixed support reaction force
S_y	Elastic limit
t	Beam thickness

T_d	Beam tip total displacement
TSP	Temperature scaling parameter
ub	Upper bound for each problem variable
ν	poisson's ratio
w	Beam width
w_{dis}	Distributed load
x_0	Starting point for the optimization variables
y_f	Displacement due to an end force load
y_w	Displacement due to a distributed load
α	Lode dependancy parameter
γ	PRBM beam subsegment length
ε	Strain
η	Beam rotation
θ	Beam tip inclination angle
Θ	Beam subsegment rotation angle
κ	PRBM beam spring stiffness
σ	Stress
σ_y	Yield strength
τ	torque
ϕ	Input force angle

1. INTRODUCTION.

Shape memory alloys (SMAs) hold a significant interest in the fields of biomedicine [1], [2] and structural engineering [3]. Their unique properties, distinct from other shape memory materials, make them attractive for various research applications. Notably, SMAs are able to recover from large deformations, resist corrosion and fatigue, exhibit high strength, and maintain biocompatibility.

Shape memory alloys are able to recover from large deflections by exploiting either the Shape Memory Effect (SME) or their Superelasticity (SE). The former is a temperature dependent behavior of SMAs that consists in the ability of the material to recover large strains induced at a low temperature by increasing the temperature of the material. The latter refers to their ability to recover large strains induced by stress at a constant high temperature by relieving the applied stress.

A critical consideration is the sensitivity of nickel-titanium (Ni-Ti) alloys (the focus of this research) to variations in their composition and thermomechanical treatments they are exposed to [4]. These variations result in diverse behaviors depending on their fabrication process. To address this, an experimental research-based process is proposed. While experimented researchers in fabrication of SMAs first define the desired characteristics of the material, then SMAs are fabricated according to the needs of each application, this is not always feasible, especially for research labs not directly involved in metallurgical engineering. This is currently the case, meaning that buying them from an outside source is the preferred option.

In various areas, Ni-Ti SMAs are preferred due to their relatively higher commercial availability and extensive research behind their behavior. The proposed methodology could potentially work with other materials as long as their behavior is comparable to Ni-Ti alloys. Ni-Ti SMAs are commercially available in different shapes, for example: tubes, sheets, foils, wires and springs. The most utilized shapes for SMA-based actuation are wires, plates and springs.

Shape memory alloy wires have the characteristic feature of functioning as artificial muscles. An example of this is shown in [5], where SMA wires are used as part of a Bio-Inspired Shape Memory Alloy Composite (BISMAL) actuator. These types of actuators offer a higher deformation in contrast to only using wires, allowing for a large stroke that is capable of generating enough thrust to move a biomimetic robotic jellyfish for autonomous underwater surveillance.

SMA wires typically only allow for strains in the range of 4% – 8%. It is possible to overcome this limitation by connecting multiple SMA wires in series to achieve larger strokes for SMA-based actuators. Furthermore, when exploiting the shape memory effect (SME), it is imperative that there is a component that strains the material once it cools down, ensuring that the material can be reset, and that the SME can take place once again to complete a cycle in any application where the SME provides actuation.

Researchers have previously ensured that the wire returns to its original position by encasing it inside a matrix that can deform as the alloy exhibits the SME, and

subsequently provide an elastic recovery that resets the alloy for the next cycle [6], [7], [8], [9], [10], [11], [12]. An example of this is shown in Figure 1-1 [7].

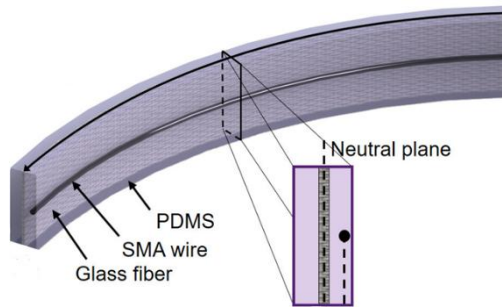


Figure 1-1.- Polydimethylsiloxane (PDMS) matrix encasing a SMA wire in order to return it to its original position upon cooling [7].

A simple but effective option is to add a bias spring to the device in order to strain the SMA upon cooling [6]. Another option is the use of a self-biased compliant structure that actively displaces the alloy in order to prepare it for the next cycle [13].

On the other hand, SMA springs have also been demonstrated to be useful in bio-inspired robots, providing high mobility to an octopus-inspired robot. The springs were selected because of their ability to provide the necessary performance in terms of stroke without consuming as much power as it would be necessary with other types of actuators. The springs were designed to deform and shape the arms of the robot, while the generation of forces was relegated to a motor driven cable system [14], [15].

Other bio-inspired robots also make use of SMA springs, such is the case of the GoQBot [16]. This caterpillar-inspired robot uses SMA springs to simulate muscle contractions. The small size of the SMA springs allows them to have a faster response time, which is a critical aspect of the robot in order to ensure dynamic stability. Similarly, the Omegabot, an inchworm-inspired crawling robot, incorporates SMA spring actuators as part of a pair of four bar mechanisms coupled by a spherical six bar mechanism. The SMA spring actuators improve the actuation speed and efficiency in cyclic actuation of the robot [17]. The primary focus of this research lies in thin plate shape memory alloys. Flat shapes are attractive for space-limited applications. Thin sheets are also machinable, an attribute that allows for the design of monolithic compliant components (Figure 1-2.a) [18]. Such is the case of flat sheet torsional SMA actuators (Figure 1-2.b), which exhibit a low profile, accompanied by high torque and energy densities without the need for auxiliary torque amplification mechanisms. The devices also show an improved response rate and power efficiency accomplished by the use of external heating elements [19]. Furthermore, it is possible to thermally train SMA plates into a desired actuated position (Figure 1-2.c) [20]. This approach uses the SME to bend or deform the material into an activated position, which can be later straightened with an antagonistically extensor actuator.

A typical drawback related to the use of SMAs as actuators is the requirement of cooling down the material previous to any attempt to apply a strain to reset it. Some of the

previously mentioned applications rely on using small actuators that cool down faster or rely on activating the actuator in a low temperature environment, for example, for deep sea exploration robotic actuators, where the surrounding water may present an optimal passive cooling environment.

A thin sheet based SMA compliant gripper has been previously developed for microscopic scale applications (Figure 1-2.d) [21]. The microgripper's performance demonstrated the promise of SMAs as gripper actuators. This work aims to extend this potential beyond the microscopic scale and establish a streamlined methodology for developing grippers with varying stroke lengths and force outputs.

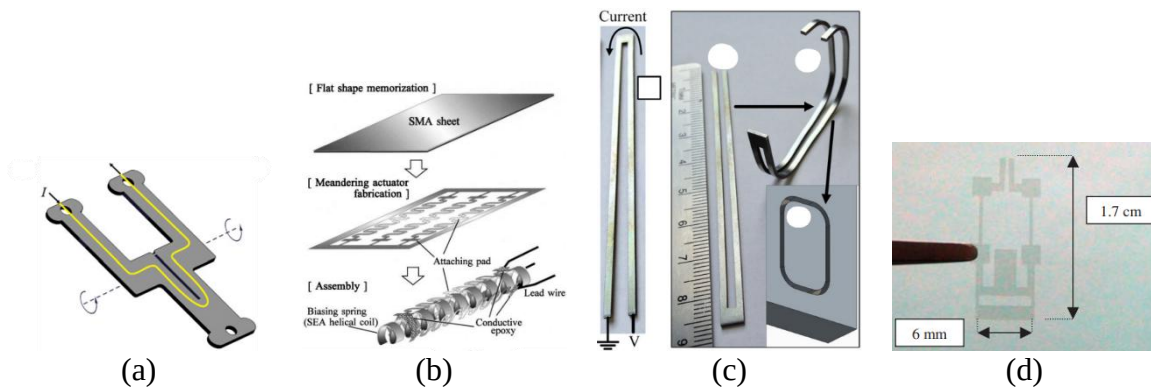


Figure 1-2.- Thin plate based SMA devices: (a) SMA-based hinge, (b) SMA torsional actuator, (c) antagonistic plate actuator and (d) microelectromechanical systems actuator [18], [19], [20], [21].

Typically, SMA wires serve as one-dimensional actuators, leveraging the SME to alter their length. However, the choice of this investigation for the design of the compliant grippers is based on SMA thin sheets, which offer a two-dimensional design space. By bending and recovering from bending, SMA sheets exert force along the bending trajectory. This provides the necessary type of trajectory required for the stroke action of a compliant gripper while also providing a force, useful for grasping objects.

1.1. JUSTIFICATION.

Researchers often tailor methodologies for creating SMA-based mechanisms in order to satisfy specific application requirements. However, this individualized approach can be time-consuming. Standardization of a method applicable to compliant grippers would speed up the design process, allowing earlier testing and greater focus in the performance optimization of the gripper.

By testing SMAs and acquiring their specific material parameters it is possible to simulate their behavior accurately. Numerical simulations will then demonstrate the functionality of our shape memory alloy-based compliant gripper. Knowing the mechanical properties of the materials is necessary to be able to create a gripper based on SMAs as the activation method. The reason for testing the materials and not relying solely on the information found on tables is that these materials are extremely sensitive to fabrication methods, thus it is necessary to measure the physical characteristics of the

materials that will be used in this particular case, otherwise the numerical simulation models could become inaccurate.

A few steps on how to create this methodology are given below, which are outlined to give a better understanding of this work:

- I. Understanding the superelasticity and shape memory effects of SMAs to be able to predict how the materials should behave.
- II. Obtaining SMAs and experimenting with them will help to better understand the previous phenomena. Their characterization is an important part of the methodology, having the correct data will greatly help the available models give better results.
- III. Some case studies should be selected to test the methodology that will be proposed through this work; they should be different enough so that they show the applicability of the method.
- IV. Mathematical and finite element models will aid in the design and analysis of the performance of the gripper. The selection of the models should be done keeping in mind the available information about the physical characteristics of the materials.
- V. The methodology should start from the material characterization to have a numerical simulation model from a cantilever beam analysis, and continue its development until the beam can be used as an actuator of a SMA compliant gripper.
- VI. In the end, the method should be well structured and follow a set of logical steps for other researchers to follow, detailing every step carefully and making it easy to follow.

1.2. GENERAL OBJECTIVE.

Develop a methodology for the synthetization of a compliant gripper using sheets made from shape memory alloy (SMA). The proposed methodology should facilitate the design process of a shape memory alloy-based compliant planar gripper to achieve the targeted displacement to deliver a force to grasp objects. The soft gripper is composed of two SMA jaws which are deformed with the help of two bias springs, then, after triggering the SME, the jaws perform the closing motion to grasp an object. Tests should be done with Ni-Ti shape memory alloys to evaluate their mechanical and thermal properties in order to create a design of a compliant gripper based on these materials.

1.3. SPECIFIC OBJECTIVES.

- To understand the superelasticity and shape memory effect phenomena and to study how to analyze large deformations of beams subject to combined loading.
- From a previous analysis of bibliography, select robust analytical models that could be used to predict the behavior of compliant mechanisms.
- Experimentation with the SMAs to quantify physical properties of the materials and have a better understanding of their behavior.

- Finite element method (FEM) modeling of a compliant gripper fabricated using SMAs and validation of results using analytical models.
- Tests and analysis of the physical design.
- Propose the methodology that allows the user to synthesize a compliant gripper, subject to specific constraints of force and stroke.

A mind map can give a better understanding of the concepts that should be understood in order to propose the methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper (Figure 1-3).

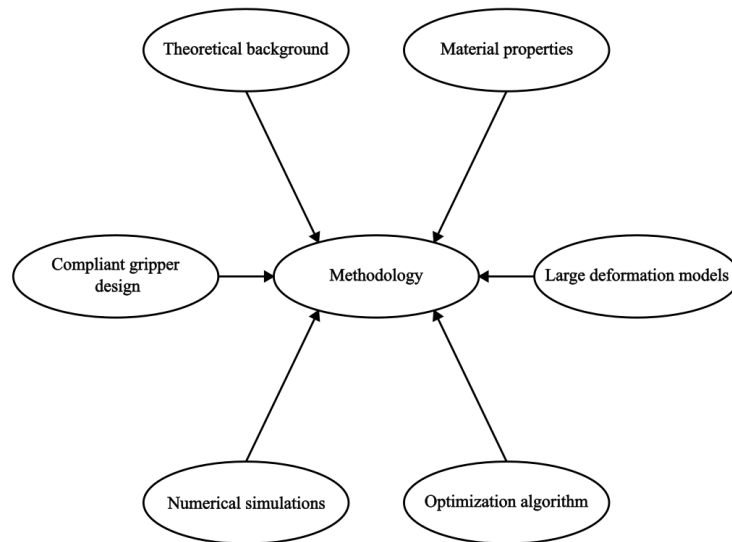


Figure 1-3.- Mind map of the required concepts to incorporate into the methodology.

As shown on Figure 1-3, the methodology should incorporate the understanding of the material behavior, this can be performing an analysis of the theoretical background of the behavior of SMAs and their use in different applications. In addition, large deformation models are useful to determine the displacement of the SMA when subject to a specific load, this can give the SMA material the required conditions to perform its task as an actuator of a compliant gripper. However, to properly calculate the displacements of SMA materials, it is important to know the specific material properties that dictate how the material deforms, once the material properties are known and the displacements can be calculated, optimization algorithms can be used to calculate the required conditions for the SMA actuator to provide a specific grasping force. Numerical simulations are useful to ensure that the compliant gripper design performs as expected, prior to its construction. These concepts are explained throughout this thesis project and the proposed methodology is presented in section 6.

1.4. ORGANIZATION OF THE FOLLOWING CHAPTERS.

In chapter 2, a theoretical background is given regarding the various tools and information required to have a better understanding of the next chapters.

In chapter 3, the pseudo-rigid body model is adapted to be used in this thesis project as a tool for the analysis of large deflection of shape memory alloy beams, in addition, the model implementation in MATLAB is explained.

In chapter 4, the numerical simulation analysis of shape memory alloy beams is performed using ANSYS Workbench, with the objective of finding the required material model specifications to successfully represent in the software the observed experimental testing results. With this model, the performance characteristics of the material can be simulated to establish performance objectives and to ensure that they can be achieved.

In chapter 5, the optimization of a cantilever beam force load and rotation is performed to achieve previously established performance objectives. The optimization algorithms *fmincon* and *ga* are paired with the pseudo-rigid body model in MATLAB using data from the numerical simulation analysis results to achieve this.

In chapter 6, the methodology is proposed and tested with two study cases, demonstrating the use of the various tools presented in previous chapters and the results are verified with experimental tests.

2. THEORETICAL BACKGROUND.

2.1. COMPLIANT MECHANISMS.

Compliant mechanisms (CMs) are defined in [22] as monolithic structures where the displacement is given by the flexibility of their structural members. SMA-based compliant mechanisms get their movement due to residual internal forces as a result of mechanical loading. Relaxation of these residual forces is done by a temperature change that enables the shape memory effect to act.

Cao [23] provides two ways of classifying CMs:

- 1) Lumped and distributed CMs: If deformation comes from localized regions (Figure 2-1.a), similar to what happens when pin joints are used in conventional mechanisms, they are called lumped compliant mechanisms. If deformation is distributed in larger regions of the mechanism instead of certain points (Figure 2-1.b), the mechanisms are known as distributed compliant mechanisms.
- 2) Partially compliant mechanisms and fully compliant mechanisms: In partially compliant mechanisms, movement is achieved using kinematic pairs along with the flexibility of the components (Figure 2-1.c). In fully compliant mechanisms, movement comes only from the deformation of their components (Figure 2-1.a and Figure 2-1.b).

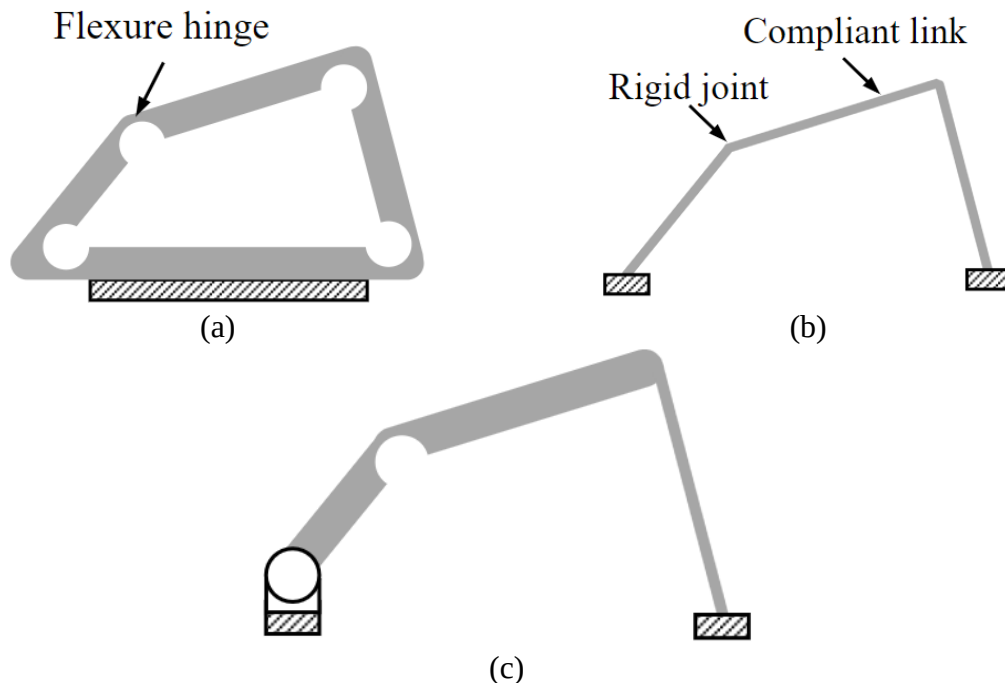


Figure 2-1.- Examples of: (a) Fully compliant lumped mechanism, (b) Fully compliant distributed mechanism and (c) Partially compliant mechanism [23].

Cao [23] also provides a comprehensive list of advantages of compliant mechanisms, which are described as follows.

Compliant mechanisms usually require less parts than a rigid body version of the same type of mechanism, even partially compliant mechanisms still hold this advantage over rigid body mechanisms, while fully compliant mechanisms do not require an assembly process at all. This is because of the reduction of kinematic pairs because of the deformation of their structure. Furthermore, a reduced number of kinematic pairs also results in reduced friction, wear and, as a consequence, the CM requires less maintenance than a rigid body mechanism. CMs store strain energy, which can be easily stored or transferred to other objects. On the other hand, since CMs rely on material deformation, they have a limited range of motion, depending on the mechanical properties of the compliant component. Fatigue failure is also a concern as the material experiences fluctuating stresses because of cyclic loading. Finally, rigid body mechanism kinematics are simpler because they primarily involve geometry. In contrast, CMs mostly require a better understanding of the forces involved and mechanical properties, as their movement stems from the deformation of the material. SMA-based compliant mechanisms combine the advantages of SMA properties with those of CMs, making them an intriguing research topic that is gaining importance. However, SMA-based CMs are not easy to classify since their deformation and recovery is thermal-based, rather than relying solely on mechanical deformation.

For a more in-depth categorization of CMs, in [24] a link is defined as the continuum connecting the mating surfaces of one or more joints. Links are divided into two classes: rigid and compliant. A compliant link may then be categorized into two other classes: simple or compound. A simple compliant link refers to one that has a constant cross section and homogeneous material properties, any link that does not cover these characteristics may be considered a compound link. Links may be thought of as comprised of only one or a combination of segments.

A SMA-based compliant link can be regarded as a compound compliant link, even if its cross-section remains constant. This is because the material properties of SMAs change based on the applied stress and temperature. Therefore, unless the material experiences a constant stress across its cross-section, it will not exhibit a homogeneous behavior. This is because the material experiences a phase transformation from twinned to detwinned martensite, depending on the amount of stress. Furthermore, if the temperature rises above the austenite phase transformation temperature A_f , the material is able to revert this phase transformation. A common metal may be able to undergo a phase transformation depending on the applied stress, however, it is unusual for a metal to be able to recover from this by simply changing its temperature. For a better understanding of this, chapter 2.2 provides a more in-depth explanation of the behavior of Shape Memory Alloys.

A SMA-based gripper may be considered as a mechanism, as defined in [24], since it couples and transforms energy, specifically for the work presented here, it couples electrical and elastic energy to produce usable movement. Additionally, the mechanism is also a compliant mechanism, since the SMA provides the movement through its recovery caused by the shape memory effect (SME). Depending on whether it incorporates rigid body links, a SMA-based gripper can be fully or partially compliant. Finally, the presence

of a SMA could categorize the mechanism as a distributed compliant mechanism, given that the material is not homogeneous, this causes the deformation to concentrate in regions of high stress.

2.2. SHAPE MEMORY ALLOYS.

For a better understanding of shape memory alloys (SMAs), their characteristics are reviewed in [4]. Their behavior is characterized by the superelasticity effect (SE), associated with the ability of the material to recover from strains at a specific temperature (Figure 2-2.a) [3]. Another important characteristic these materials have is called the shape memory effect (SME) characterized by the ability to eliminate residual strains originated from cyclic loading by going from a low to a high temperature (Figure 2-2.b) [3]. SMAs have four important temperatures that allow them to present their characteristic behaviors. The most common SMAs are called type 1 and present the following temperatures in increasing order: M_f, M_s, A_s, A_f , where M refers to the martensitic phase, A refers to the austenitic phase, f refers to the final temperature required to reach a specific phase while s refers to the start of the phase transformation process. This transformation between phases is responsible for the SME. If the temperature is lower than M_f , by applying stress reorientation of martensite crystals allows the material to obtain detwinned martensite from twinned martensite, unloading leaves behind a significative amount of residual stress. To be able to restore the twinned martensite it is necessary to transform the material to the austenitic phase, increasing the temperature to A_f , after that it is possible to obtain twinned martensite by decreasing the temperature.

The SE effect is due to the transformation of twinned martensite when loading at a temperature higher than A_f , the effect starts with elastic austenite, then the transformation to detwinned martensite starts at a critical stress value. When transformation ends, elastic loading of martensite starts, unloading the elastic martensite starts the reverse transformation because at this range of temperatures only austenite is stable in the stress-free state. When unloading is done, the SMA presents practically zero residual strains. These alloys can generally be based on Ni-Ti and Cu, cold worked and annealed Ni-Ti alloys are considered best for the majority of engineering applications due to their superior superelastic properties, stability against temperature variations and resistance to corrosion and fatigue.

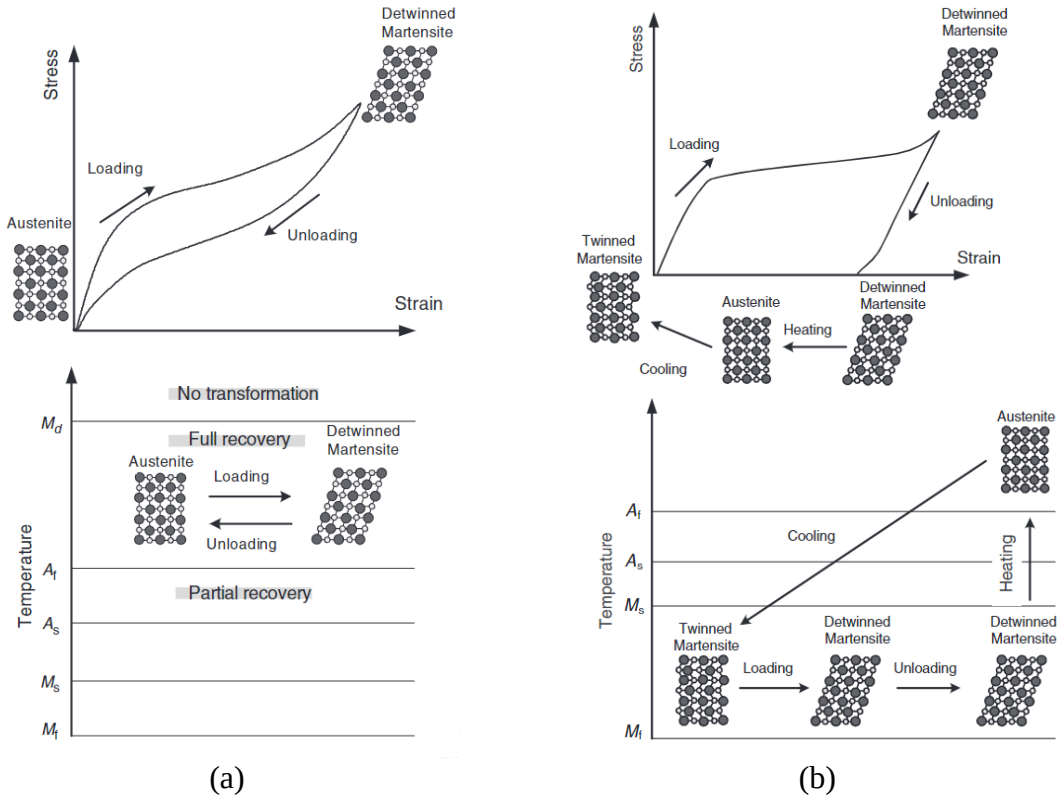


Figure 2-2.- Stress-strain curve for: (a) superelasticity and (b) shape memory effect [3].

A SMA-based compliant gripper requires a load to induce a transformation from twinned martensite to detwinned martensite. Since this transformation depends on the induced stress, it will typically not be produced throughout the entire link of a compliant mechanism, instead, it will concentrate on the same regions where the stress is concentrated. For example, consider the behavior of a cantilever beam. In an end-force-loaded cantilever beam, the stress is mostly concentrated in the area closer to the fixed support, this would mean that this area would experience a transformation from twinned martensite to detwinned martensite, while the other end could potentially still be in the twinned martensite state. In between these regions there would be a combination of both phases (Figure 2-3). Furthermore, the area transformed into detwinned martensite would be responsible of the recovery of the alloy when a phase change due to an increase in temperature takes place, even if increasing the temperature of the beam creates a phase change from martensite to austenite in the entire beam, only the region where detwinned martensite was created would have an effect on the shape recovery of the beam.

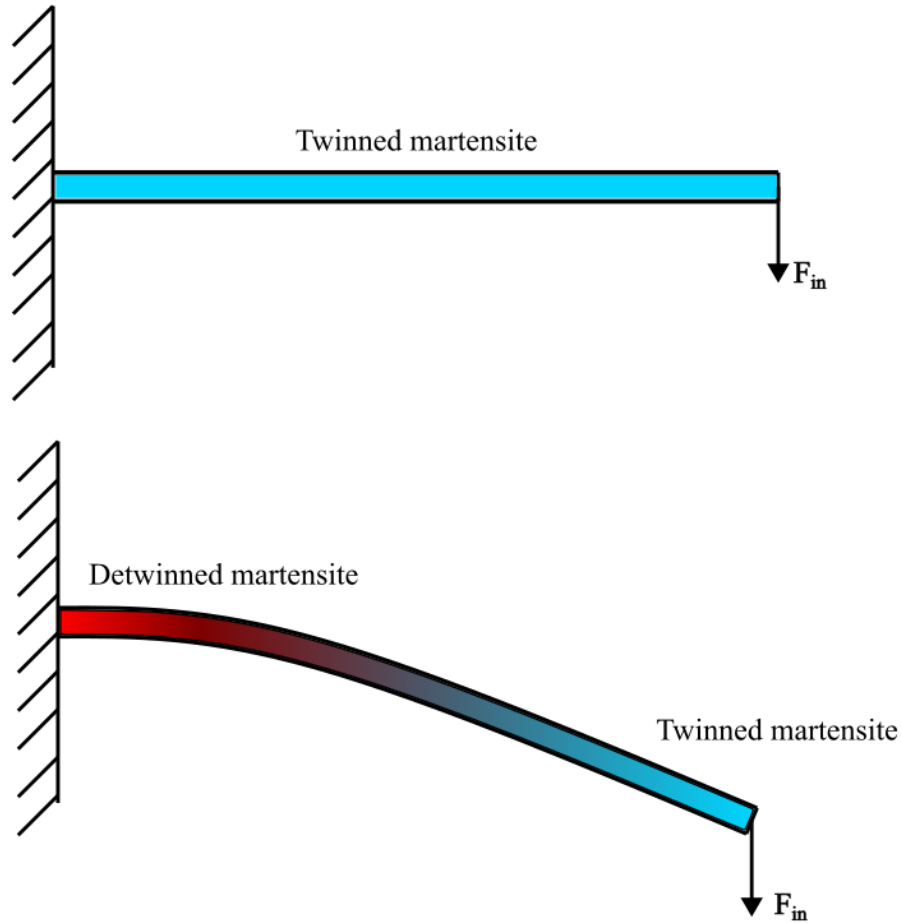


Figure 2-3.- Shape memory alloy end-loaded cantilever beam phase transformation caused by the applied load.

At temperatures equal or lower than M_s , the beam is in martensite phase across its entire cross section, for temperatures above M_s , detwinned martensite can be stress induced. This means that loading will allow the material to experience an increase in strain at a constant stress. Unloading allows the material to revert this strain not due to an increase in temperature but because of the relaxation of stress [25]. The elastic modulus of the material is not easily calculated at this temperature range since it becomes temperature and strain dependent.

The shape memory effect is predominantly unidirectional, meaning that when the temperature of the material falls below M_s , external strain must be applied to induce a new shape change. Shape memory alloys are typically assumed to behave the same in tension and compression [4].

Shape memory alloys are materials known for their remarkable resistance to corrosion, excellent biocompatibility and high resistance to bending. These properties make them an attractive research focus as actuators across various areas of engineering.

2.3. APPLICATIONS OF SMAS.

The great potential of SMAs has led to exploring their use in various research fields as described below.

Their application in structural engineering is reported in [3], noting that most SMA applications in seismic response control usually involve the superelastic effect, instead of relying on active control of the material temperature. This choice is based on the unique re-centering ability and the capacity of superelastic alloys to dissipate energy. Furthermore, SMAs are able to withstand large deformations and have an excellent resistance to fatigue and corrosion.

The research done in [3] shows the use of SMA wires as part of bracing systems, by using pre-tensioned wires above their A_f temperature, provoking a superelastic behavior. These wires allow the devices that they are a part of to have a re-centering capability.

Shape memory alloy wires have been tested in damper devices because of their excellent energy dissipation capacity (Figure 2-4.a) [26]. SMAs are also present in hybrid devices where the alloys are used because of their re-centering capabilities while other materials, such as steel struts provide energy absorption (Figure 2-4.b) [27]. Ni-Ti springs are able to deliver re-centering and damping capabilities to seismic retrofitting devices, while Ni-Ti Belleville washers offer good energy dissipation (Figure 2-4.c) [28].

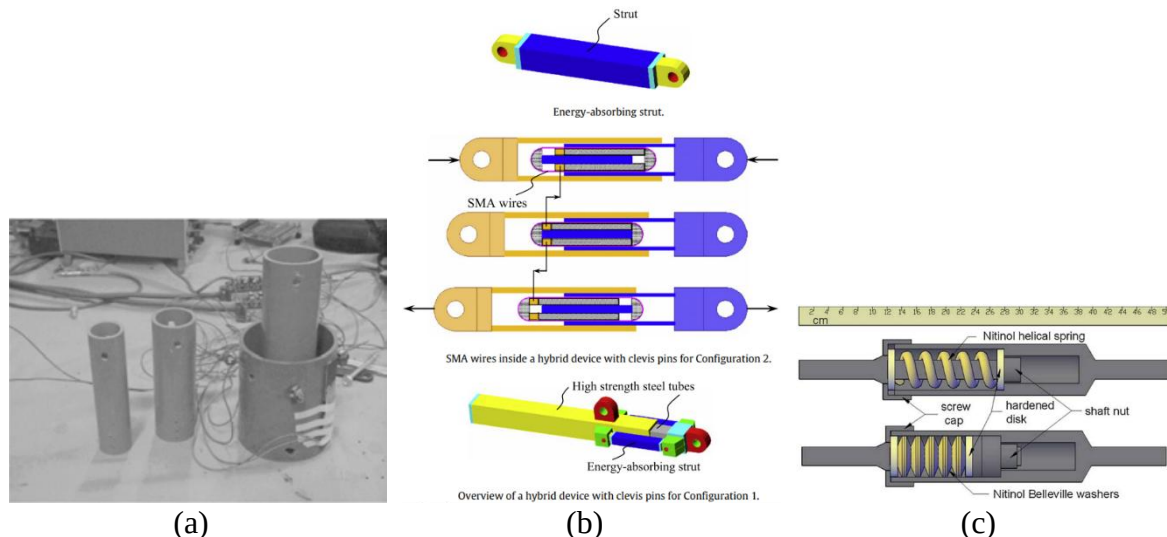


Figure 2-4.- Shape memory alloy devices for seismic response control: (a) Damper devices, (b) energy-absorbing strut and (c) tension-compression device [26], [27], [28].

The research of shape memory alloy-based seismic response control devices has shown that Ni-Ti SMAs are well-suited for seismic response. However, the high material costs act as a barrier to their widespread adoption in this field. The quantities of material required for seismic applications are typically much higher than those needed in other areas, such as biomedicine and aerospace industries.

The biomedical field benefits from the SMA pseudoelastic effects, in [1] a detailed review of the use of the Ni-Ti shape memory alloy shows how it has been an important part of the orthodontic field since its first implementation back in 1975. This implementation uses a Ni-Ti alloy wire with an austenite finish temperature (A_f) equal to the average temperature of the buccal cavity, exploiting the large and constant force of the wires to help brackets generate wide dental movements. Orthodontic devices also make use of the SME, since the wires are in martensitic phase when inserted into the buccal cavity, then deformed into the brackets, to then transform into the austenite phase whenever the patient ingests hot food or drinks. Since recovery is constrained, the wires are able to exert light forces on the teeth as long as the temperature is above A_f (Figure 2-5.a) [1]. Orthodontic distractors (Figure 2-5.b) [29] and palatal arches (Figure 2-5.c) [1] also exploit the pseudoelastic properties of SMAs, such as being able to apply constant forces without another type of actuator. Endodontic files fabricated using Ni-Ti SMAs (Figure 2-5.d) [1] also benefit from the pseudoelastic behavior of the material since it increases their flexibility, recovery of deformation and limits the applied force when paired with a rotating motor.

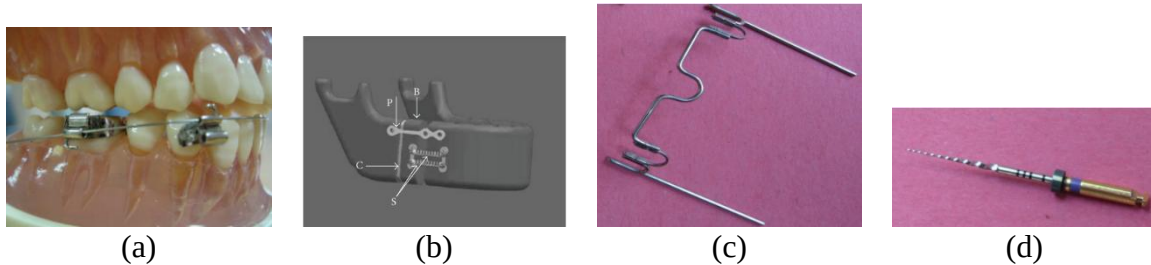


Figure 2-5.- Orthodontic and endodontic applications of Ni-Ti shape memory alloys: (a) orthodontic wire, (b) orthodontic distractor, (c) palatal arch and (d) endodontic file [1], [29].

A review of medical applications of shape memory alloys is presented in [2]. It states the importance of the biocompatibility of the materials, since it allows them to remain inside of the body without producing negative reactions or releasing undesired substances into the bloodstream. This property is mainly inherited by the titanium, since its oxidation produces a protective layer that also renders it harmless to the human body while highly increasing its resistance to corrosion.

Several examples are found in the literature of the use of shape memory alloys in the cardiovascular research field. The Simon filter is the first shape memory alloy device developed in this field, which exploits the SME to filter blood clots inside the bloodstream. This is accomplished by deforming the filter in its martensitic phase and placing it inside a catheter tip. A saline solution maintains the low temperature until the filter is positioned inside of the body. When the filter is released by the catheter, the body temperature allows the shape memory effect to take place, and the filter returns to its undeformed shape (Figure 2-6.a) [2].

A similarly functioning device that exploits the SME is called the atrial septal occlusion device. The atrial hole splits the upper part of the heart into the right and left atria. An

abnormally open atrial hole may reduce life expectancy. The atrial septal occlusion device uses SMA wires and a waterproof film of polyurethane to seal the hole when placed in the atrial hole with the help of a catheter (Figure 2-6.b) [2].

Self-expandable stents are used in vascular applications, to provide support to several types of tubular passages inside of the body (Figure 2-6.c) [2]. These stents are similarly deployed and expanded inside the body as the Simon filter and consist of a cylindrical shape memory alloy scaffold structure.

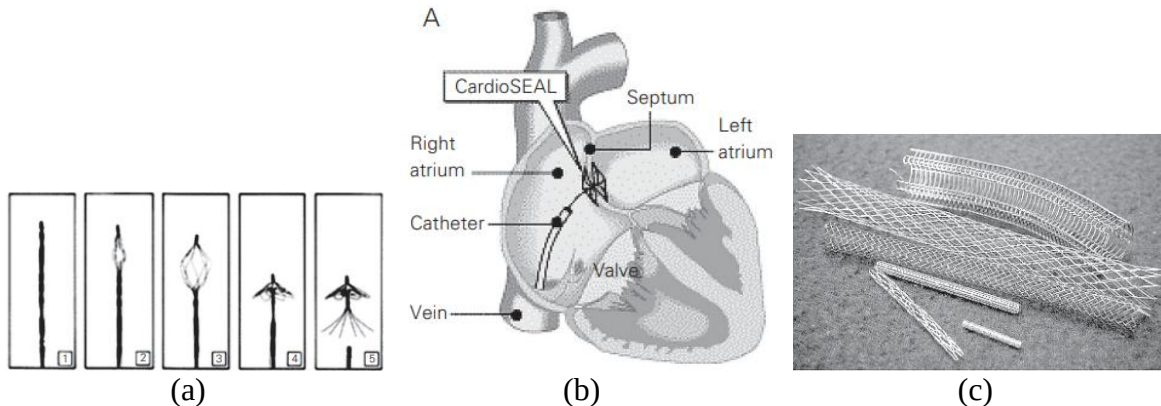


Figure 2-6.- Vascular applications of Ni-Ti shape memory alloys: (a) Simon filter, (b) atrial septal occlusion device and (c) stents [2].

The orthopedic field benefits from the shape memory effect by exploiting the stress generated during constrained shape recovery. Orthopedic staples (Figure 2-7.a) [1] and plates (Figure 2-7.b) [1] are deformed in martensitic phase. The constant stress generated by the constrained recovery allows the device to join two fractured pieces together. Similarly, Ni-Ti rods are incorporated into devices that help to correct scoliosis (Figure 2-7.c) [30].

Vertebrae spacers (Figure 2-7.d) [1] rely on the high deformability in its martensitic phase of SMAs to facilitate the positioning in place between the vertebrae. The device is able to recover its original shape when the temperature increases inside of the human body.

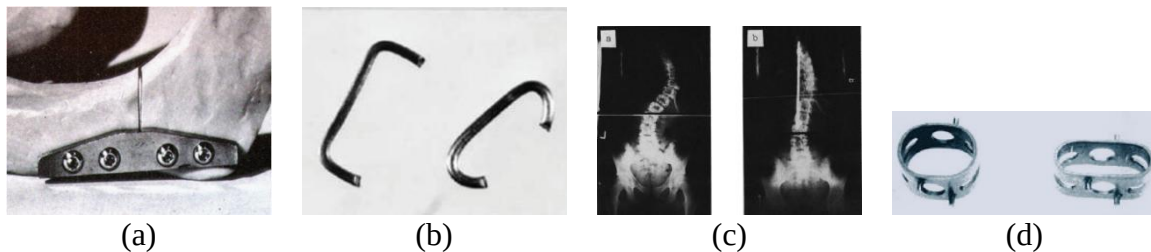


Figure 2-7.- Orthopedic applications of Ni-Ti shape memory alloys: (a) Mandibular fracture plate, (b) SMA staples, (c) SMA rods and (d) vertebrae spacers [1] [2], [30].

The various characteristic performance attributes found in Ni-Ti SMAs are reviewed in [31]. SMAs are preferred over other materials in the medical field due to their deployment capabilities, as mentioned in some previous examples, allowing various

devices to be passed through very small catheters and then spring back to their original shape. This is possible either by exploiting the elastic deployment due to the SE of an austenitic SMA or by using the SME of the material by deforming the device in a martensitic phase only for it to recover its original shape when increasing its temperature inside the body.

SMA's also prevent strain localization by creating a more uniform strain when compared to other materials, allowing them to be used as guidewires for the deployment of other devices.

Ni-Ti alloys are also biomechanically compatible, since they are compliant in response to pressure from surrounding tissue, unlike metals such as steel and titanium which are stiff relative to biological tissue.

The force applied by Ni-Ti SMA's is determined by temperature. Since the body remains at a relatively constant internal temperature, Ni-Ti devices are able to deliver constant stress that is useful for various purposes from orthodontic to vascular applications.

The long-range expansion forces delivered by Ni-Ti stents allow them to apply outward force until they reach their design diameter. In contrast, stainless steel stents would require an expansion above the specified diameter to account for spring back effects. This oversized diameter may harm the vessels and create more complications. The superelastic hysteresis also provides stents with the ability to deliver outward force to vessel walls and a high resistance to crushing.

The above-mentioned features among others make Ni-Ti alloys a desirable material to work in various medical applications. Their potential as actuators of a compliant gripper based on their many positive attributes shows that it is possible to incorporate them into devices capable of applying large forces and displacements, features required to complete the closing and opening tasks of a compliant gripper.

The design of Ni-Ti based stents involving Ni-Ti thin-films is explored in [32]. Using origami inspired structures it is possible to place implants inside of the human body, inserting a small, crumpled thin film with a specific design inside a catheter. The thin film is crumpled in its martensitic phase at cryogenic temperatures. After being introduced inside the body, its expansion would start since the material is austenitic at room temperature, similar to how self-expandable stents work. An example of this is demonstrated Figure 2-8 [32] where the transformation temperatures were increased from those described above to simplify their testing.

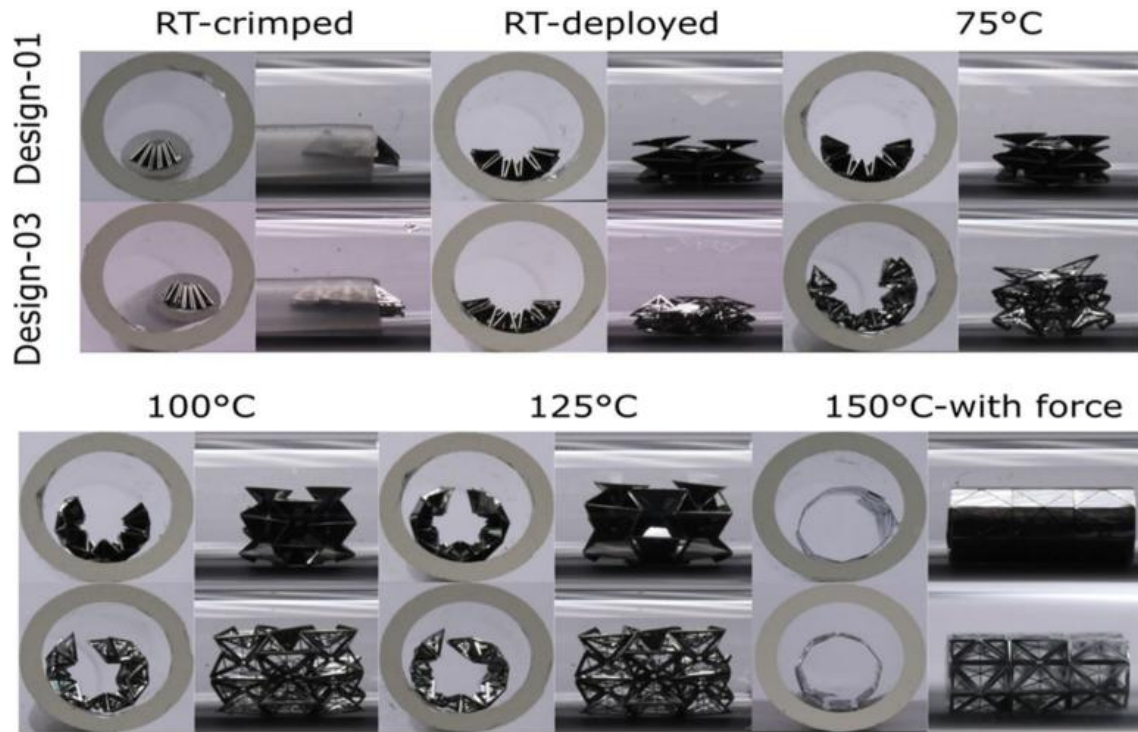


Figure 2-8.- Origami-inspired thin film shape memory alloy devices [32].

To date, the majority of investigated applications involving shape memory alloy devices rely on activating the shape memory effect at ambient room temperature. This approach is particularly advantageous in medical contexts, where the body temperature consistently surpasses this threshold. However, when considering the shape memory effect as an actuation method for a compliant gripper, it becomes essential that the austenite phase starting temperatures (A_s) exceed room temperature. This preventive measure ensures that the gripper does not inadvertently actuate during undesirable moments.

2.4. SME TRIGGERING.

Triggering either by SE or SME requires variations in temperature (Figure 2-9), and any means to increase the temperature is valid as long as they do not damage the material.

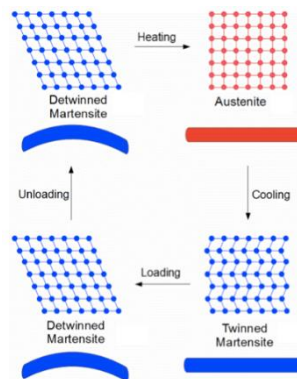


Figure 2-9.- Shape memory effect and phase transformation due to heating and cooling.

Sometimes it is desirable for the workplace temperature to trigger the effect, if this is the case, instead of heating the material one may wish to refrigerate it when the effect is not needed, then stop refrigerating when it is necessary to trigger the effect. It is important to note that, while not necessary, having a smooth transition from low to high temperature is better for most applications. In recent projects some workarounds to achieve a uniform transition throughout the material have been made.

An electric energy supply system is proposed in [33] to address the problem of having excess temperatures that could damage the alloy and its functionality.

Shape memory polymers are actuated using an oven in [34] with the objective of having uniform heating. This allows the triggering of the SME of origami inspired structures and, in turn, the unfolding of the structures is more controlled, as the oven increases its temperature to 130 °C, temperature at which these polymers transform to their memorized shape. Using an oven is impractical for this project, since the oven simulates a SME trigger due to the ambient temperature, which as previously stated, is not this case.

Due to the extremely sensitive nature of SMAs to their fabrication processes, transformation temperatures may vary, along with their elastic modulus and some other important characteristics. In this project the means of reaching the transformation temperature are not the main focus, so it is not necessary to have a highly controlled method to increase the temperature of the material, then using a resistive material, a heat gun or a small current passing through the material is enough as long as the material is able to reach its transformation temperature (A_f).

2.5. LARGE DEFLECTION OF BEAMS.

Large deflection theories are typically used to analyze the displacement of beams subject to end loads of forces and moments.

Being able to calculate the displacement as a consequence of a load is important for this work since a SMA-based gripper should be able to open and close after triggering the SME. The open position will be defined by a displacement given by an end load and the closed position will be reached after increasing the temperature of the material. Being able to predict the displacement of the beams that form the gripper jaws is essential in order to use them as actuators.

The work done in [35] is one of the first iterations regarding large deflection of beams, where deflections are analytically calculated for end loaded cantilever beams. This is the basis of how these types of analysis should be solved when the load is defined by a force on the free end of the cantilever beam, then an approximation of its displacement using elliptic integrals is made, where the approximation error goes up to 0.5%. The use of elliptic integrals avoids non linearities in the analytic solution. Nonetheless, the lack of moment loads at the free end of the beam makes these solutions incomplete, but it is a good starting point to understand the expected behavior of a beam subject to end loads.

The analysis incorporating a moment load to the free end is done by Su where large beam deflections are analyzed with a combined free end force and moment load (Figure 2-10) [36].

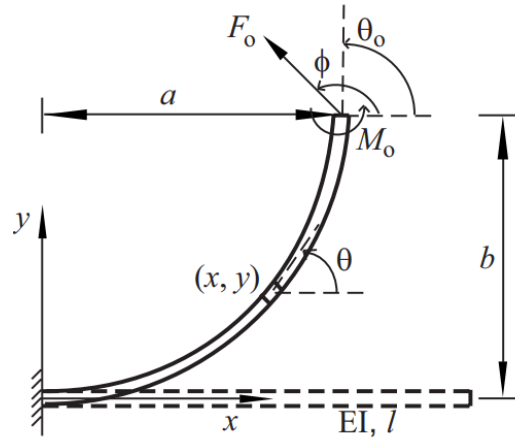


Figure 2-10.- Large beam deflection of a cantilever beam with combined end load [36].

The analytical equations describing this phenomenon are given as follows, which can be utilized to calculate the normalized horizontal and vertical tip positions $\frac{a}{l}$ and $\frac{b}{l}$:

$$\sqrt{\alpha} = \frac{1}{2} \int_0^{\theta_0} \frac{d\theta}{\sqrt{[\cos(\theta_0 - \phi) - \cos(\theta - \phi)] + \kappa}} \quad (2-1)$$

$$\frac{a}{l} = \frac{1}{2\sqrt{\alpha}} \int_0^{\theta_0} \frac{\cos \theta d\theta}{\sqrt{[\cos(\theta_0 - \phi) - \cos(\theta - \phi)] + \kappa}} \quad (2-2)$$

$$\frac{b}{l} = \frac{1}{2\sqrt{\alpha}} \int_0^{\theta_0} \frac{\sin \theta d\theta}{\sqrt{[\cos(\theta_0 - \phi) - \cos(\theta - \phi)] + \kappa}} \quad (2-3)$$

Where:

$$\alpha = \frac{F_0 l^2}{2EI}, \kappa = \frac{\beta^2}{4\alpha}, \beta = \frac{M_0 l}{EI} \quad (2-4)$$

This beam analysis method requires the following variables: elastic modulus E , dimensional properties such as the cross-section moment of inertia I and the length of the beam l , as well as the loading parameters: force magnitude F_0 , force angle ϕ and the moment magnitude M_0 , the force index α , and load ratio κ along with the tip slope angle θ_0 .

The use of elliptic integrals makes these equations hard to solve, in addition to this, they are hard to modify to better suit the purpose of this project. An easier to grasp model is shown in the next section.

2.6. PSEUDO-RIGID BODY MODEL.

Analytical equations describing large deflections are often complicated, various models have been created to facilitate their analysis, such is the case of the work done by Su where large deflection of beams are modeled using the pseudo-rigid body model with 3 revolute joints (PRBM 3R) as it allows the analysis of a beam as a series of rigid bodies with revolute joints that allow it to deflect (Figure 2-11) [36]. The revolute joints allow the rigid links to move, and each deflection contributes to the total deflection of the beam. A single beam could represent the behavior of a single gripper jaw. Furthermore, the model is simple enough that it can be modified according to the necessities of this project.

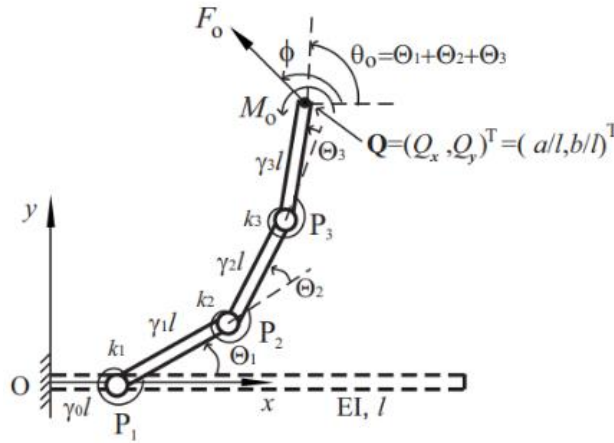


Figure 2-11.- Pseudo-rigid body 3R model of a cantilever beam subject to a combined end load [36].

The analytical equations (2-1) to (2-4) make use of variables related to the geometric properties of the beam and the loading parameters to calculate the horizontal position a/l , vertical position b/l and a given end slope θ_0 . The latter being problematic as it is one of the integration limits of the analytical equations and, in many cases, an unknown variable. This is presumably one of the reasons why researchers often avoid the analytical equations and decide to work using simplified models. The equations used for this thesis project should serve the purpose of being able to calculate a specific displacement, given by the position of the tip, and should only depend on known variables, such as the elastic modulus of the beam E and its geometric properties, namely the length of the beam l and the cross-section moment of inertia I . This is the case for the equations given by Su [36], which will be presented below.

In order to have a more precise model, the PRBM 3R is analyzed by Chen [37], where the optimal parameters of subsegment length γ_i and spring stiffness κ_{θ_i} were found using a particle swarm optimizer as: $\gamma_0 = 0.125$, $\gamma_1 = 0.35$, $\gamma_2 = 0.388$, $\gamma_3 = 0.136$, $\kappa_{\theta_1} = 3.25$, $\kappa_{\theta_2} = 2.84$, and $\kappa_{\theta_3} = 2.95$. These values for γ_i and κ_{θ_i} work for various types of end loads.

The static equations used in [36] are used to find the tip position and end slope when subjected to a certain moment M and force F load. These equations are shown below. Given the loading parameters F , φ and M , the torques at each pin joint can be calculated using the following equation:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} -\gamma_1 s_1 - \gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_1 c_1 + \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_3 s_{123} & \gamma_3 c_{123} & 1 \end{bmatrix} \begin{bmatrix} F \cos \varphi * l \\ F \sin \varphi * l \\ M \end{bmatrix} \quad (2-5)$$

Where:

$$c_1 = \cos(\Theta_1), s_1 = \sin(\Theta_1) \quad (2-6)$$

$$c_{12} = \cos(\Theta_1 + \Theta_2), s_{12} = \sin(\Theta_1 + \Theta_2) \quad (2-7)$$

$$c_{123} = \cos(\Theta_1 + \Theta_2 + \Theta_3), s_{123} = \sin(\Theta_1 + \Theta_2 + \Theta_3) \quad (2-8)$$

Equations (2-5) to (2-8) are obtained from the forward kinematics equations. The angles Θ_1 , Θ_2 and Θ_3 are the angles of each subsegment of the PRBM, see Figure 2-11. Variables γ_i represent the length of each subsegment of the PRBM and they can be fine-tuned to have better results depending on the load type.

Assuming the spring torques are proportional to the PRB angles:

$$\tau_i = \kappa_i \Theta_i, i = 1, 2, 3 \quad (2-9)$$

The spring stiffness can be normalized as a variable that relates the spring stiffness coefficient with the beam's Young's modulus, its moment of inertia and its length:

$$\kappa_i = \kappa_{\Theta_i} \left(\frac{EI}{l} \right), i = 1, 2, 3 \quad (2-10)$$

Substituting equations (2-9) and (2-10) into equation (2-5), the normalized non-dimensional statics equation is obtained as:

$$\begin{bmatrix} \kappa_1 \Theta_1 \\ \kappa_2 \Theta_2 \\ \kappa_3 \Theta_3 \end{bmatrix} = \begin{bmatrix} -\gamma_1 s_1 - \gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_1 c_1 + \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_3 s_{123} & \gamma_3 c_{123} & 1 \end{bmatrix} \begin{bmatrix} F \cos \varphi * l \\ F \sin \varphi * l \\ M \end{bmatrix} \quad (2-11)$$

Using equation (2-11), it is possible to calculate the angles that allow a cantilever beam defined by its material properties and geometry to reach a certain tip position. This position is expressed as the horizontal position Q_x , vertical position Q_y and tip slope angle θ_0 and can be calculated using forward kinematics as shown below:

$$Q_x = \gamma_0 + \gamma_1 c_1 + \gamma_2 c_{12} + \gamma_3 c_{123} \quad (2-12)$$

$$Q_y = \gamma_1 s_1 + \gamma_2 s_{12} + \gamma_3 s_{123} \quad (2-13)$$

$$\theta_0 = \Theta_1 + \Theta_2 + \Theta_3 \quad (2-14)$$

Equations (2-11) to (2-14) will ultimately help calculate the required load to properly position the jaws of the gripper in order to have a specified output force when using the SME, this will be done by using optimization methods to test different configurations until a minimum error is reached.

2.7. OTHER DEVELOPMENTS IN COMPLIANT MECHANISM MODELLING.

Wang et al. [38], studied the analytical modeling of serial flexure-based compliant mechanisms (CMs), including cases where external loads are present not only at the end of flexible members but can also be somewhere in the middle of the member itself. These problems are solved using matrix-based models. The matrix-based models are well developed, however, the PRBM is more suited for prototype analysis of compliant mechanisms due to its simpler nature.

Another important development in the analysis of CMs is presented by Yu et al. [39], as it is mentioned that PRBMs are not able to describe axial deformation when only torsional springs are used. Thus, a PRR PRMB is proposed, where the beam's axial deformation is represented by a prismatic pair with a compression spring and the deflection of the beam is described by the two revolute joints in this prismatic-revolute-revolute (PRR) configuration (Figure 2-12) [39].

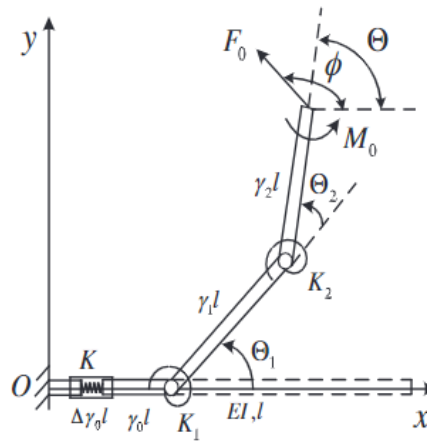


Figure 2-12.- PRR PRBM of a cantilever beam subject to a combined end load [39].

This model can be considered as an upgrade of the PRBM shown in section 2.6, nevertheless, the PRBM 3R is preferred since the SMA actuators will be subject to bending loads in order to deform them to be able to exploit their SME, for this reason, the effect of an axial load is not required.

2.8. TARGET SHAPE OPTIMIZATION USING SMAS.

With the PRBM it is possible to calculate the displacement of a beam subject to a combined end load. In order to design the gripper jaws, it is necessary to find the required load that can deform thin SMA sheets into the required shape for them to be used as actuators.

An interesting methodology is used in [40], where target shape optimization of superelastic SMA CMs is done by optimizing the geometry and mechanical properties of the SMAs. This is accomplished with additive manufacturing to create the Ni-Ti alloys better suited for a particular mechanism. The difference with this thesis project is that the methodology to be created should use commercially available SMAs, instead of SMAs fabricated in situ, as additive manufacturing is not readily available in every research center, even less when working with metal alloys. Similarly, Jovanova and Frecker [41] separate the target shape optimization into two phases. In the first phase an approximation of the CM (Figure 2-13.a) [40] is made by using a method similar to the PRBM proposed in [36], where low deformation sections are transformed into rigid links and high deformation sections are transformed into springs (Figure 2-13.b) [40].

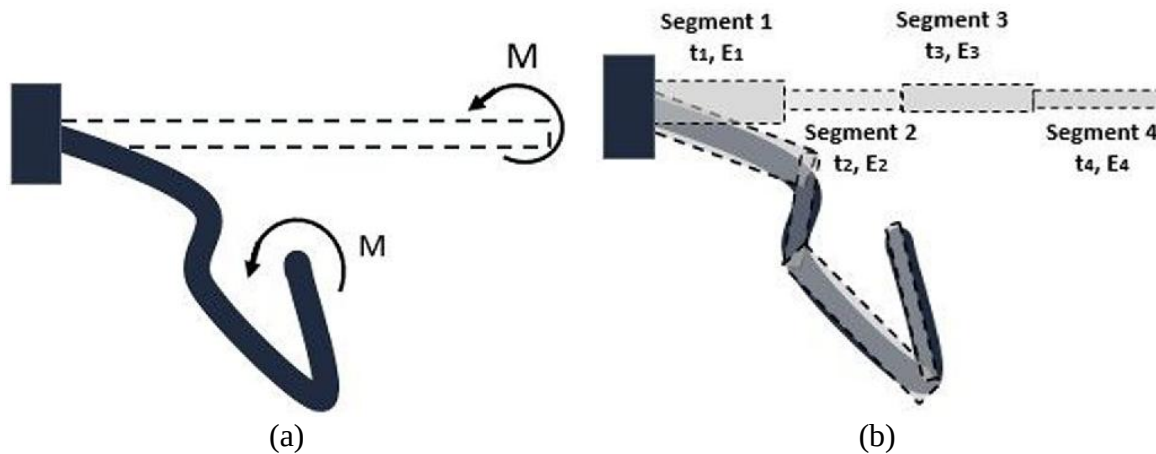


Figure 2-13.- Target shape optimization: (a) original shape and (b) discretization of original shape [40].

This gives an early visualization of the CM where, in the second phase, rigid links are replaced by low elasticity materials and springs are replaced by superelastic material. These characteristics are given to the material using additive manufacturing processes. The first phase is a good reference point for a tested methodology that, with some modifications, could be generalized to work in this project where additive manufacturing is not an option.

2.9. OPTIMIZATION PROBLEM FORMULATION.

Optimization problems are focused on finding the best solution to a problem subject to different constraints.

In this project, the optimization problem is centered on calculating the required end load of a cantilever beam to deform a SMA-based actuator in such a way that allows the actuator to exert a force onto an object, as a consequence of the activation of the SME. Optimization problems should be formulated as shown below:

Minimize:

$$f(a) = \left| \frac{x_{expected} - x(a)_{current}}{x_{expected}} \right| * 100\% \quad (2-15)$$

Subj. To:

$$\begin{aligned} g_1: a_i &\geq a_{min} \\ &\vdots \\ g_n: a_n &\geq a_{min} \end{aligned} \quad (2-16)$$

$$\begin{aligned} h_1: a_i &\leq a_{max} \\ &\vdots \\ h_n: a_n &\leq a_{max} \end{aligned} \quad (2-17)$$

In equation (2-15), f is the optimization problem, in this case, it represents the absolute percentage error calculation of the expected value of variable x , compared to its current value. In this equation, when both variables have the same value, f becomes 0%, meaning that the error has been reduced to 0%. Variable x is dependent on the value of variables $a_1, a_2, \dots, a_n$, the constraints for these variables are defined by equations (2-16) and (2-17). These constraints define the lower and upper limit for each optimization variable ($a_1, a_2, \dots, a_n$). These constraints are not arbitrary, they depend on the real limitations given by the equipment and materials available.

In section 5.3 the optimization problem is defined for this particular project.

2.10. MIXED VARIABLE OPTIMIZATION.

Variables of an optimization problem are classified in [42] into two main categories: continuous and discrete. The former refers to the variables commonly used in ideal problems, these can be any real number and usually their limits are problem constraints, if these variables are unconstrained, these variables can take any value and be represented by: $-\infty \leq x \leq \infty$. The latter category refers to a type of variable commonly used in real life problems, as their values are constrained to fixed values and are generally limited to upper and lower limits, for example, bolt dimensions, pipe sizes, beam profile dimensions, etc. Two more types of variables can be added to this classification as a subtype of discrete variables:

Integer variables are a type of discrete variable that can only represent integer values. The second type of discrete variables are binary variables, they can only represent values of 0 or 1, useful to determine the existence or lack of an element and are commonly used in programming. Problems that can contain combinations of these types of variables are known as mixed variable problems. For this project, variables related to the material may be considered discrete variables, since they depend on the material itself and the processes to which they will be exposed for the research of this project. Variables related to the loads that will affect the shape of the material may be considered continuous, since the applied force is easier to modify than the geometric properties of the material. Still, an important part of the methodology is to define which variables should be part of the optimization problem and which ones are better off being considered as constants.

Multi-variable optimization problems (MV-OPT) can be classified as shown next:

- MV-OPT 1: problem functions are twice continuously differentiable. Discrete variables can have non-discrete values during the solution process.
- MV-OPT 2: problem functions are not differentiable at least at some points of the feasible set. Discrete variables can have non-discrete values during the solution process.
- MV-OPT 3: problem functions may not be differentiable. Some discrete variables cannot have non discrete values during the solution process. Some problem functions can only be evaluated using discrete variable values during the solution process.
- MV-OPT 4: problem functions may not be differentiable. Some variables are linked to others, this means that assigning a value to one variable specifies values for others.
- MV-OPT 5: also known as combinatorial problems, these are purely discrete and non-differentiable problems. An example of these types of problems is the traveling salesman problem.

As an example, thickness depends on commercial availability of the material, then, in [42] a better representation of equations (2-16) and (2-17) is given:

$$t_i \in t_{available}, t_{available} = (t_1, t_2 \dots t_n) \quad (2-18)$$

Meaning that t can take any value from vector $t_{available}$, the number of values t_n depends on the variety of sizes acquired for the project.

From the classification of mixed variable optimization problems, this particular problem belongs to type 1 (MV-OPT 1). In this project, the optimization problem is focused on calculating the required loading conditions of a cantilever beam based on a required displacement to deliver a specified output force using the SME. The discrete variables that define the input load conditions can have non-discrete values during the solution process.

2.11. OPTIMIZATION METHODS.

Solution algorithms for MV-OPT 1 types of problems are well documented and tested, some of the most common solution algorithms are:

Branch and bound (BBM): In [42] two types of BBM methods are given, the first consists of solving the problem using only discrete variables during the solution process, the second one consists of solving the problem by finding a continuous solution that can then lead to a discrete solution. A better explanation of how they work is given below.

Basic branch and bound: In this method all possible combinations of discrete values are evaluated and then the one with the best objective function value that does not violate any constraints is selected. Some criteria can be added to avoid the necessity of looking at all possible combinations if the behavior of the functions is known and predictable, for example, if increasing x_1 is known to increase the cost function, when all constraints are

satisfied it is not necessary to keep increasing its value as it will always lead to a higher cost. This method can take considerably longer to find the solution when the number of variables increases.

Branch and bound with local minimization: In this method a continuous solution is given to the problem regardless of the discrete variables, only considering their upper and lower limits, then, the branch and bound method is applied using only the discrete values closest to the continuous solution and the best solution that does not violate any constraints is chosen.

Genetic algorithms (GA): These methods are specially interesting, and they are also known as evolutionary methods or nature inspired methods. These methods are based on heuristic and stochastic rules. They do not require continuity or differentiability of the objective function. They are applicable to a large number of problems. Since they are highly robust, they are ideal to solve the type of problem of this thesis project. The disadvantage of these methods is that they may need a large number of calculations even for reasonably sized problems. Another disadvantage of these methods is that it is uncertain that the global minimum has been reached.

2.12. COMPARISON OF AVAILABLE OPTIMIZATION METHODS.

Some other optimization methods are available, however, the previously mentioned methods are applicable to various types of problems, while other algorithms may be more specialized towards a specific type of minimization objective. The disadvantage of the robustness of these methods is the higher computational power required to get a solution for a problem.

These methods and some others are compared in Table 2-1 [42].

Method	MV-OPT problem type solved	Can find feasible discrete solution?	Can find global minimum for convex problem?	Need gradients?
Branch and bound	1–5	Yes	Yes	No/Yes
Simulated annealing	1–5	Yes	Yes	No
Genetic algorithm	1–5	Yes	Yes	No
Sequential linearization	1	Yes	Yes	Yes
Dynamic round-off	1	Yes	No guarantee	Yes
Neighborhood search	1	Yes	Yes	Yes

Table 2-1.- Comparison of various methods applicable to mixed variable optimization problems [42].

The most robust methods are the BBM, Simulated Annealing (SA) and GA since they can be used to solve every type of multi-variable optimization problem. BBM and GA will be used in this project due to their availability and easy implementation in MATLAB.

2.13. LINKED VARIABLES.

It is hard to guarantee that a problem will always be a Type 1 MV-OPT problem, since geometric property-related variables are actually linked to other variables such as the Young's modulus or the transformation temperature of the material. In these cases, geometric properties and thermo-mechanical properties are considered to be linked variables. In ideal cases a commercially available material should be available in different sizes while maintaining the same properties, however, this is not always the case, due to the sensitivity of the material to manufacturing processes. Having linked variables in a problem changes the type of the problem from a Type 1 to a Type 4 MV-OPT problem, and the last three methods shown in Table 2-1 cannot be used anymore. For this reason, the SMA actuators should come from a single sheet of material, to ensure that the mechanical properties are the same, ensuring that the problem remains as a Type 1 MV-OPT problem.

2.14. OPTIMIZATION METHOD IMPLEMENTATION.

Optimization problems can be solved using MATLAB functions included in the global optimization toolbox.

If the objective function is non-linear, functions such as *fmincon* and *ga* are the best alternative to solve these types of problems. These functions are used to minimize a problem function. The BBM method is related to *fmincon*, which requires the following syntax:

$$x = \text{fmincon}(\text{fun}, x_0, A_{ineq}, b_{ineq}, A_{eq}, b_{eq}, lb, ub, \text{nonlcon}, \text{options})$$

Where:

fun: objective function of the optimization problem.

x_0 : starting point for optimization variables.

A_{ineq} : inequality constraints coefficients matrix.

b_{ineq} : inequality constraints results vector.

A_{eq} : equality constraints coefficients matrix.

b_{eq} : equality constraints results vector.

lb: lower bound for each problem variable.

ub: upper bound for each problem variable.

nonlcon: non-linear constraints of the problem.

The genetic algorithms are related to the MATLAB function *ga*, it is worth mentioning that *ga* in the most recent version of MATLAB (R2022a) does not allow the use of integer variable constraints at the same time as non-linear equality constraints, only non-linear inequality constraints.

This method can be used using the following syntax:

$$x = \text{ga}(\text{fun}, n_{vars}, A_{ineq}, b_{ineq}, A_{eq}, b_{eq}, lb, ub, \text{nonlcon}, \text{intcon}, \text{options})$$

Where:

fun: objective function of the optimization problem.

x_0 : starting point for optimization variables.

A_{ineq} : inequality constraints coefficients matrix.

b_{ineq} : inequality constraints results vector.

A_{eq} : equality constraints coefficients matrix.

b_{eq} : equality constraints results vector.

lb : lower bound for each problem variable.

ub : upper bound for each problem variable.

nonlcon: non-linear constraints of the problem.

n_{vars} : number of optimization variables.

nonlcon: non-linear problem constraints.

In MATLAB, problems are specified using the following structure:

$$\min f(x) \text{ subject to } \begin{cases} c(x) \leq 0 \\ c_{eq}(x) = 0 \\ A_{ineq} * x \leq b_{ineq} \\ A_{eq} * x = b_{eq} \\ lb \leq x \leq ub \end{cases}$$

Both methods, *fmincon* and *ga*, have their own limitations that will make them more useful than the other depending on the problem at hand.

3. THE PSEUDO-RIGID BODY MODEL AS A TOOL IN THE ANALYSIS OF LARGE DEFORMATIONS OF BEAMS.

3.1. THE PSEUDO-RIGID BODY MODEL (PRBM).

The pseudo-rigid body model with 3 revolute joints (PRBM 3R) has been selected to represent the mechanical behavior of the gripper jaws, this is because it is a well-developed model and thus there is a sufficient amount of information about it, in addition, it is a simple mathematical model that can be modified in order to better suit the needs of this project. The equations (2-6) to (2-8) and equation (2-11) are the basis of the model. It is possible to implement this model using MATLAB, where the optimization algorithms are also available, meaning that they can be interconnected in such a way that the PRBM becomes part of the optimization problem formulation. To prove that the results are correct, a cantilever beam is programmed into ANSYS using the same material and geometric properties that will be input into the PRBM 3R model.

For this particular model, some variables are given that could be fine-tuned to have better results, these variables are shown in Figure 2-11, being $\kappa_1, \kappa_2, \kappa_3, \gamma_0, \gamma_1, \gamma_2$ and γ_3 , these have an important role in equation (2-11) and they dictate how accurate the PRBM results are. Chen et al. [37], have worked on obtaining the optimal parameters to better suit three cases: pure force, pure moment and combined tip loading. For this work, the parameter values given for combined loading are used in all cases, since they could be applied to different types of loads without sacrificing too much accuracy.

The optimal parameters used have the following values: $\gamma_0 = 0.125$, $\gamma_1 = 0.35$, $\gamma_2 = 0.388$, $\gamma_3 = 0.136$, $\kappa_{\theta_1} = 3.25$, $\kappa_{\theta_2} = 2.84$, and $\kappa_{\theta_3} = 2.95$.

The properties of the beam are: $l = 100$ mm, $w = 10$ mm, $t = 1$ mm and $E = 41$ GPa.

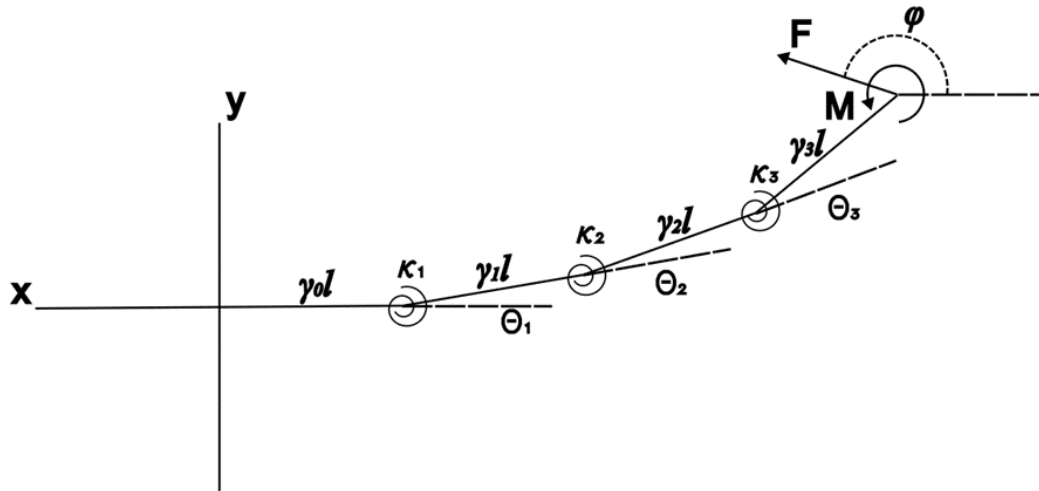


Figure 3-1.- Cantilever beam subject to a combined end load.

The cantilever beam is represented as shown in Figure 3-1. The results can be divided into three cases: pure force, pure moment and combined loading. Table 3-1 shows the individual results for vertical and horizontal tip positions after loading calculated using

analytic equations (2-1) to (2-4), PRBM 3R equations (2-11) to (2-14) and ANSYS simulation results.

Tip position results after loading								
Pure force			Analytic equations		PRBM 3R		ANSYS	
Force (N)	Force angle (Degrees)	Moment (Nm)	a (mm)	b (mm)	Qx (mm)	Qy (mm)	X (mm)	Y (mm)
12	30°	0	94.80	24.40	96.10	25.10	96.15	25.32
15	120°	0	44.90	77.90	42.60	78.80	45.82	78.15
20	220°	0	17.80	-82.60	13.70	-82.70	18.87	-82.94
Pure moment			Analytic equations		PRBM 3R		ANSYS	
Force (N)	Force angle (Degrees)	Moment (Nm)	a (mm)	b (mm)	Qx (mm)	Qy (mm)	X (mm)	Y (mm)
0	0	0.05 CCW	99.50	7.30	99.50	7.30	99.65	7.24
0	0	0.1 CCW	98.50	14.50	98.50	14.50	98.60	14.40
0	0	0.25 CW	91.20	-34.90	91.10	-34.90	91.45	-34.65
Combined loading			Analytic equations		PRBM 3R		ANSYS	
Force (N)	Force angle (Degrees)	Moment (Nm)	a (mm)	b (mm)	Qx (mm)	Qy (mm)	X (mm)	Y (mm)
12	30°	0.05 CCW	93.60	27.10	95.10	27.80	95.27	27.75
15	120°	0.1 CCW	39.30	78.90	36.90	79.90	40.12	79.43
20	220°	0.25 CW	7.30	-79.70	2.80	-79.90	7.90	-80.63

Table 3-1.- Tip position results after loading using: Analytic equations, PRBM 3R equations and ANSYS simulation.

Total tip deflection can be obtained from the previous table to have a more condensed view of the results of each model, as shown in Table 3-2. To obtain the total tip deflection T_d , in ANSYS it should be added to the results list, to obtain it in the MATLAB code, it is simply calculated as $T_d = \sqrt{a^2 + b^2}$ for analytic equations and $T_d = \sqrt{Q_x^2 + Q_y^2}$ for the PRBM results.

Total tip deflection results					
Pure force			Analytic equations	PRBM 3R	ANSYS
Force (N)	Force angle (Degrees)	Moment (Nm)	Total deflection (mm)	Total deflection (mm)	Total deflection (mm)
12	30°	0	24.95	25.40	25.62
15	120°	0	95.42	97.49	95.19
20	220°	0	116.53	119.53	116.36
Pure moment			Analytic equations	PRBM 3R	ANSYS
Force (N)	Force angle (Degrees)	Moment (Nm)	Total deflection (mm)	Total deflection (mm)	Total deflection (mm)
0	0	0.05 CCW	7.32	7.32	7.25
0	0	0.1 CCW	14.58	14.58	14.47
0	0	0.25 CW	35.99	36.02	35.69
Combined loading			Analytic equations	PRBM 3R	ANSYS
Force (N)	Force angle (Degrees)	Moment (Nm)	Total deflection (mm)	Total deflection (mm)	Total deflection (mm)
12	30°	0.05 CCW	27.85	28.23	28.15
15	120°	0.1 CCW	99.55	101.81	99.47
20	220°	0.25 CW	122.25	125.82	122.39

Table 3-2.- Total deflection results using: Analytic equations, PRBM 3R equations and ANSYS simulation.

Before doing experimental tests, it is expected that ANSYS simulation results are closer to reality since the models used by the program have been rigorously chosen among various options to have results as precise as possible. Comparing the two other models with ANSYS is a fair comparison since ANSYS should deliver the most accurate results. This information is summarized in Table 3-3 calculating the percentage error.

Tip deflection model results compared with ANSYS simulation results				
Pure force			Analytic equations	PRBM 3R
Force (N)	Force angle (Degrees)	Moment (Nm)	%e	%e
12	30°	0	2.60	0.83
15	120°	0	0.24	2.42
20	220°	0	0.15	2.72
Pure moment			Analytic equations	PRBM 3R
Force (N)	Force angle (Degrees)	Moment (Nm)	%e	%e
0	0	0.05 CCW	0.92	0.92
0	0	0.10 CCW	0.73	0.73
0	0	0.25 CW	0.85	0.92
Combined loading			Analytic equations	PRBM 3R
Force (N)	Force angle (Degrees)	Moment (Nm)	%e	%e
12	30°	0.05 CCW	1.07	0.29
15	120°	0.10 CCW	0.08	2.35
20	220°	0.25 CW	0.11	2.81
			Average error	Average error
			0.75	1.55

Table 3-3.- Tip deflection error calculation results for analytic equations and PRBM tip position results compared to ANSYS simulation results.

As shown in Table 3-3, the PRBM 3R has a higher error than the analytic equations, this error is still acceptable as it is only 1.55% in average, the maximum error obtained was 2.8%, still being acceptable for most applications.

3.2. CANTILEVER BEAMS IN SERIES.

As seen in the previous subsection, the PRBM model can describe the large deflection of a beam with a fairly low error margin. Nonetheless, it is necessary to make some modifications to be able to use it for shape optimization. To this end, some additions must be made, and their results are compared with other models, specifically FEM models as there is currently no experimental data to compare to. For this comparison, an arrangement of three beams connected in series using fixed joints is proposed. Each beam has the following dimensions: length = 100 mm, width = 10mm and thickness = 0.16 mm, this is shown in Figure 3-2. The thickness dimension was chosen based on available SMA sheets for this project, in addition, this dimension is the most problematic in FEM since the material is very thin and undergoes large deformations with little load.

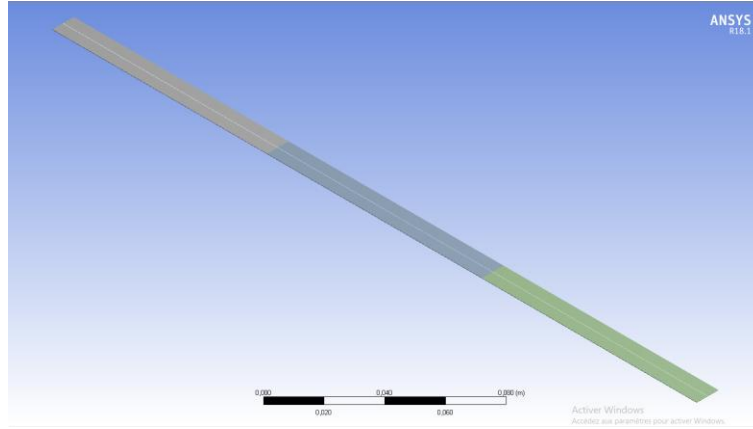


Figure 3-2.- Arrangement of three cantilever beams used to compare results between PRBM and FEM results.

The first version of the PRBM program using MATLAB, based on equation (2-11) was lacking in its formulation as it assumes that there is a single beam, making it impossible to modify the dimensions and material properties of the material to add the information of the other beams. In later chapters, being able to modify the material properties or dimensions of each beam will be useful to emulate the behavior of a SMA. Next, a better model that is not constrained by simplifications of dimensions and material properties will be developed. Some equations are expanded so it is easier to code, this is shown in equation (3-1).

$$\begin{bmatrix} \kappa_1 \theta_1 \frac{EI}{l} \\ \kappa_2 \theta_2 \frac{EI}{l} \\ \kappa_3 \theta_3 \frac{EI}{l} \end{bmatrix} = \begin{bmatrix} -\gamma_1 s_1 - \gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_1 c_1 + \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_2 s_{12} - \gamma_3 s_{123} & \gamma_2 c_{12} + \gamma_3 c_{123} & 1 \\ -\gamma_3 s_{123} & \gamma_3 c_{123} & 1 \end{bmatrix} \begin{bmatrix} F \cos(\phi) * l \\ F \sin(\phi) * l \\ M \end{bmatrix} \quad (3-1)$$

The model is still the same, it yields the same results, but this arrangement of variables is far simpler to code and adapt to add more beams in series. Now the values of the moment of inertia I , length l and Young's modulus E of each beam can be saved in an array and used to calculate the displacement of each beam. Expanding this equation into being usable with multiple beams connected as mentioned before requires applying the same methodology that can be inferred from how the equation works.

The interpretation of the sines and cosines, represented by s_i and c_i in equation (3-1), is that the force of the n th torsion spring is affected by the angle of all segments that come before it (from the load to the spring). This is more noticeable when a second beam is added as shown in Figure 3-3.

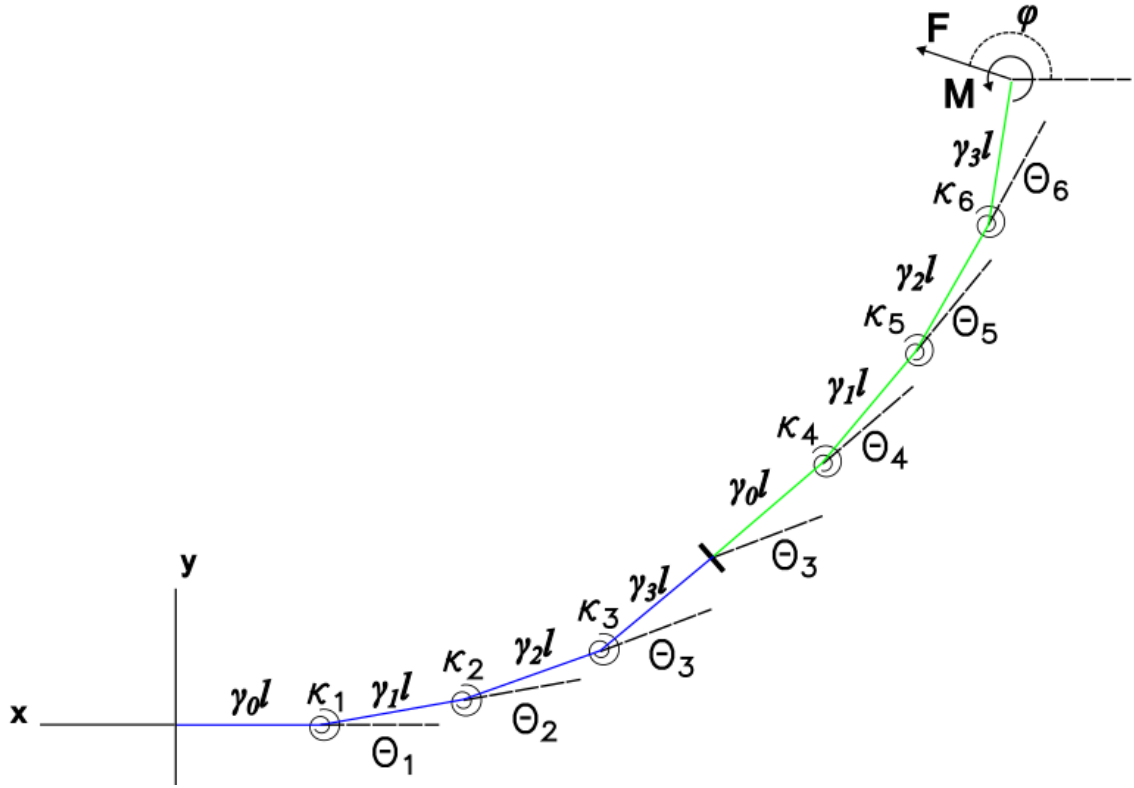


Figure 3-3.- PRBM of two cantilever beams connected by a fixed joint.

In the figure, for example, a torsion spring κ_n receives a moment due to the moment load but also because of the force load. This is expected as the force creates a moment given by the distance between the force and the spring and the angles $\Theta_1, \Theta_2, \dots, \Theta_6$ which affect the force components in X and Y. The length of the segment is given by l , referring to the total length of the beam, multiplied by a factor γ , these factors are given by [37] and are optimized to give better results when the beam is subjected to combined end loads. The angle of deflection of each spring has to be multiplied by its segment length as well as adding the distances and deflections of all springs affecting its moment, for a better explanation, consider Figure 3-4.

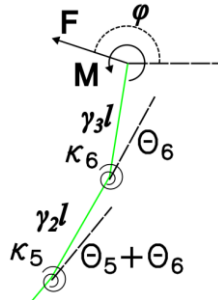


Figure 3-4.- Example of moment produced in spring k_5 by force F and moment M .

In this example, torsion spring κ_5 receives a moment given by the X and Y components of force F in addition to moment M , using a counterclockwise rotation as positive, the following are the three moment loads it receives:

Moment due to force $F_x = F \cos(\phi) (-\gamma_3 l \sin(\theta_6) - \gamma_2 l \sin(\theta_5 + \theta_6))$

Moment due to force $F_y = F \sin(\phi) (\gamma_3 l \cos(\theta_6) + \gamma_2 l \cos(\theta_5 + \theta_6))$

Moment due to moment load M

These moments need to be summed, and their result should be equals to the stiffness of spring κ_5 , multiplied by its angle Θ_5 and the product of $\frac{EI}{l}$ representing the spring torques. This is the case for all springs, thus, the further a spring is from F , more terms must be added to this equation, this can be generalized into equation (3-2) where beam n of a series of m beams is being analyzed.

$$\begin{bmatrix} \kappa_1 \Theta_{3n-2} \frac{EI}{l} \\ \kappa_2 \Theta_{3n-1} \frac{EI}{l} \\ \kappa_3 \Theta_{3n} \frac{EI}{l} \end{bmatrix} = SM1 * \begin{bmatrix} F \cos(\phi) * l \\ F \sin(\phi) * l \\ M \end{bmatrix} \quad (3-2)$$

Where:

$$SM1 = \begin{bmatrix} -\gamma_1 S_{1...(3n-2)} - \gamma_2 S_{1...(3n-1)} - \gamma_3 S_{(1...3n)} + DY & \gamma_1 C_{1...(3n-2)} + \gamma_2 C_{1...(3n-1)} + \gamma_3 C_{1...(3n)} + DX & 1 \\ -\gamma_2 S_{1...(3n-1)} - \gamma_3 S_{(1...3n)} + DY & \gamma_2 C_{1...(3n-1)} + \gamma_3 C_{1...(3n)} + DX & 1 \\ -\gamma_3 S_{1...(3n)} + DY & \gamma_3 C_{1...(3n)} + DX & 1 \end{bmatrix} \quad (3-3)$$

Here, equation (3-2) is further modified to include the effects of the deflection of other beams between the tip of beam n and the load at the tip of beam m . For example, when analyzing beam 1 (blue), DX and DY represent the entire deflection of beam 2 (green) as shown in Figure 3-5, included in equation (3-3), in this case $n = 1$.

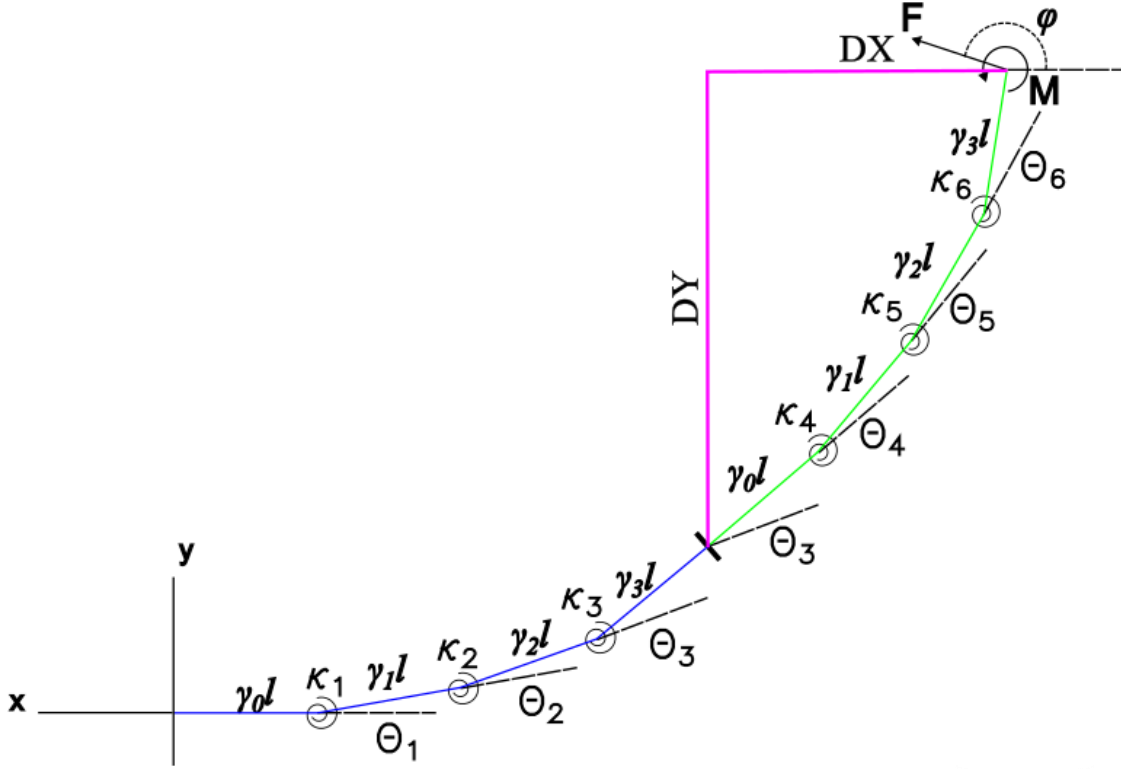


Figure 3-5.- Meaning of deflection variables DX (deflection X) and DY (deflection Y) for a two-beam arrangement.

For this example, these distances are:

$$\begin{aligned} DX &= \gamma_0 l_{c_{1...3}} + \gamma_1 l_{c_{1...4}} + \gamma_2 l_{c_{1...5}} + \gamma_3 l_{c_{1...6}} \\ DY &= -\gamma_0 l_{s_{1...3}} - \gamma_1 l_{s_{1...4}} - \gamma_2 l_{s_{1...5}} - \gamma_3 l_{s_{1...6}} \end{aligned} \quad (3-4)$$

The terms in DY are negative because the Y component of F causes the beams to rotate in a clockwise orientation whenever φ is positive. This equation can be generalized to apply to any beam n from a series of m beams as equations (3-5) and (3-6).

$$DX = \sum_{i=n+1}^m (\gamma_0 l_{c_{1...(3(i-1))}} + \gamma_1 l_{c_{1...(3i-2)}} + \gamma_2 l_{c_{1...(3i-1)}} + \gamma_3 l_{c_{1...(3i)}}) \quad (3-5)$$

$$DY = \sum_{i=n+1}^m (-\gamma_0 l_{s_{1...(3(i-1))}} - \gamma_1 l_{s_{1...(3i-2)}} - \gamma_2 l_{s_{1...(3i-1)}} - \gamma_3 l_{s_{1...(3i)}}) \quad (3-6)$$

With these additions, the model now considers the effects of a different beam into the deflection calculations. To consider different beam properties, one only needs to change the value of E , I and L depending on the beam that is being analyzed.

3.3. CANTILEVER BEAMS IN SERIES WITH INTERMEDIATE LOADS.

To further complement the PRBM, adding intermediate loads in-between beams could be useful if a specific gripper jaw shape is not possible. Thus, the PRBM has to adapt by

adding these loads and calculating their effects on the overall shape of the arrangement. Figure 3-6 shows this new implementation of the PRBM.

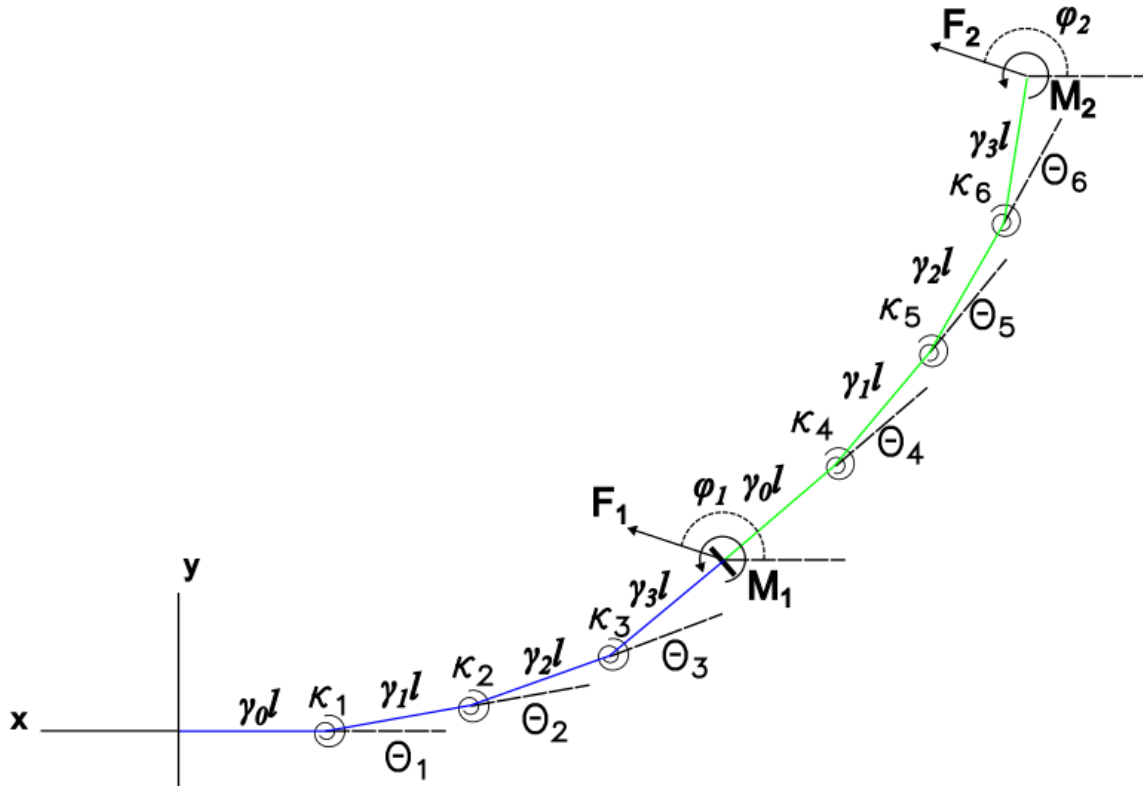


Figure 3-6.- Arrangement of two beams with intermediate loads using the PRBM.

In this new model, each beam is analyzed in three steps.

Step 1.

The moment due to moment loads at any spring of the n th beam from a series of m beams is equal to the sum of all moments from M_n to M_m . This can be calculated using equation (3-7).

$$M_{nth} = \sum_{i=n}^m M_i \quad (3-7)$$

Step 2.

The moment at each of the three springs of the n th beam due to its own force F_n can be calculated with a slightly modified version of equations (3-2) and (3-3). This is represented in equations (3-8) and (3-9).

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = SM2 * \begin{bmatrix} F_n \cos(\phi_n) * l \\ F_n \sin(\phi_n) * l \end{bmatrix} \quad (3-8)$$

Where:

$$SM2 = \begin{bmatrix} -\gamma_1 S_{1...(3n-2)} - \gamma_2 S_{1...(3n-1)} - \gamma_3 S_{1...(3n)} & \gamma_1 C_{1...(3n-2)} + \gamma_2 C_{1...(3n-1)} + \gamma_3 C_{1...(3n)} \\ -\gamma_2 S_{1...(3n-1)} - \gamma_3 S_{1...(3n)} & \gamma_2 C_{1...(3n-1)} + \gamma_3 C_{1...(3n)} \\ -\gamma_3 S_{1...(3n)} & \gamma_3 C_{1...(3n)} \end{bmatrix} \quad (3-9)$$

Step 3.

The moment at each of the three springs of the n th beam due to forces F_{n+1} to F_m can be calculated using variables DX and DY . For this to work, these variables are modified when considering a force from a different beam. Figure 3-7 helps to better explain how this works.

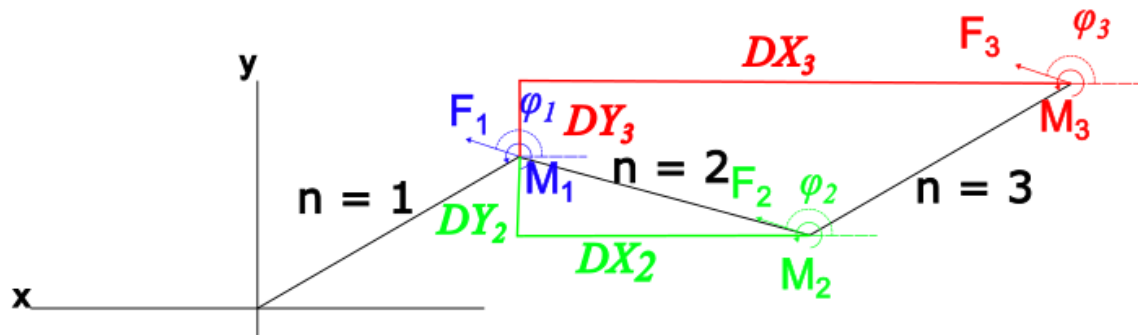


Figure 3-7.- Analysis of Beam1 and the effects of forces at the tips of Beam 2 (green) and Beam 3 (red).

Consider the analysis of Beam 1 from Figure 3-7. In step 1, the effects of M_1 to M_3 are considered. In step 2 the effects of load F_1 are also considered. Force F_3 and its angle φ_3 provoke components Fx_3 and Fy_3 . This also happens with F_2 and φ_2 , creating components Fx_2 and Fy_2 . Figure 3-7 shows a beam that has already been deformed. Beams 1 to 3 are not necessarily straight, but it simplifies the visualization of the effects that loads F_2 and F_3 have on Beam 1. It is easy to see that there are four more moments to consider:

- Fx_3 creates a moment when multiplied by distance DY_3 .
- Fy_3 creates a moment when multiplied by distance DX_3 .
- Fx_2 creates a moment when multiplied by distance DY_2 .
- Fy_2 creates a moment when multiplied by distance DX_2 .

Distances DX_3, DY_3, DX_2 and DY_2 can be calculated using the same principle as equations (3-5) and (3-6). This applies for all subsequent beams from the one that is being analyzed until reaching the tip of the series arrangement. One must keep in mind that these distances are then added to each spring distance from its tip. This can be generalized to analyzing any beam n of a series of m beams in equations (3-10) and (3-11).

$$\begin{bmatrix} D \\ E \\ F \end{bmatrix} = \sum_{i=n+1}^m [SM3] * \begin{bmatrix} F_i \cos(\phi_i) * l \\ F_i \sin(\phi_i) * l \end{bmatrix} \quad (3-10)$$

Where:

$$SM3 = \begin{bmatrix} -\gamma_1 s_{1...(3n-2)} - \gamma_2 s_{1...(3n-1)} - \gamma_3 s_{1...(3n)} + DY_i & \gamma_1 c_{1...(3n-2)} + \gamma_2 c_{1...(3n-1)} + \gamma_3 c_{1...(3n)} + DX_i \\ -\gamma_2 s_{1...(3n-1)} - \gamma_3 s_{1...(3n)} + DY_i & \gamma_2 c_{1...(3n-1)} + \gamma_3 c_{1...(3n)} + DX_i \\ -\gamma_3 s_{1...(3n)} + DY_i & \gamma_3 c_{1...(3n)} + DX_i \end{bmatrix} \quad (3-11)$$

Where the deflections can be calculated using equation (3-12).

$$DX_i = \sum_{j=n+1}^i (\gamma_0 l c_{1...(3(j-1))} + \gamma_1 l c_{1...(3j-2)} + \gamma_2 l c_{1...(3j-1)} + \gamma_3 l c_{1...(3j)}) \quad (3-12)$$

$$DY_i = \sum_{j=n+1}^i (-\gamma_0 l s_{1...(3(j-1))} - \gamma_1 l s_{1...(3j-2)} - \gamma_2 l s_{1...(3j-1)} - \gamma_3 l s_{1...(3j)})$$

The final equation, considering all three steps can be summarized in equation (3-13). Here, the n th beam of a series of m beams is being analyzed. As a final addition, equation (3-13) should include the specific elastic modulus E and geometric properties I and l of the n th beam.

$$\begin{bmatrix} \kappa_1 \Theta_{(3n-2)} \frac{EI}{l} \\ \kappa_2 \Theta_{(3n-1)} \frac{EI}{l} \\ \kappa_3 \Theta_{(3n)} \frac{EI}{l} \end{bmatrix} = \begin{bmatrix} M_{nth} \\ M_{nth} \\ M_{nth} \end{bmatrix} + \begin{bmatrix} A \\ B \\ C \end{bmatrix} + \begin{bmatrix} D \\ E \\ F \end{bmatrix} \quad (3-13)$$

As shown, equation (3-13) is a set of three equations, each beam should be used to generate a set of 3 equations, resulting in a system of $3m$ equations with $3m$ unknown variables ($\Theta_1, \Theta_2, \dots, \Theta_{3m}$). These equations are easy to implement into a mathematical software, such as MATLAB.

3.4. MATLAB CODE.

MATLAB is a programming language that allows the user to perform various types of calculations. It is used on this project because, apart from being able to solve the PRBM once all the equations are written as code, it also incorporates optimization algorithms such as BBM and GA, which will be used in later sections alongside the PRBM to calculate the required load to deform a SMA material in order to use the SME.

Equation (3-13) can be coded into MATLAB, although it is not as straightforward. Since equation (3-13) is basically three equations, one for each spring, and one set of three for

each beam, the number of equations that must be coded will be $3n$. Writing each equation one by one is easy, but inefficient, so the program needs to do this automatically. One needs to keep in mind that these equations have multiple unknown variables, being the angles of deflection of each beam, that accumulate the further a torsion spring is from the load. A flow diagram of how the code works is given in Figure 3-8, it is based on equations (3-7) to (3-13).

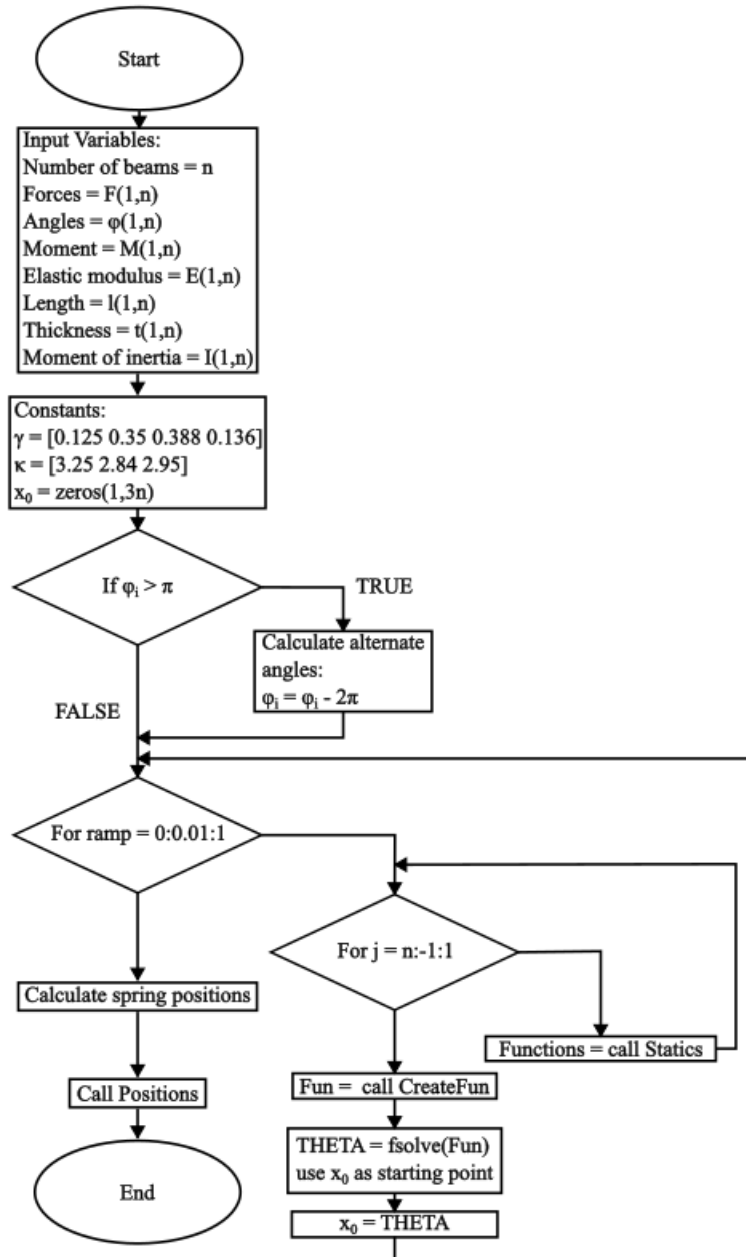


Figure 3-8.- Flow diagram for PRBM main program MATLAB code.

As shown in Figure 3-8, the program works by first introducing the input variables required to solve equations (3-7) to (3-13). In addition, the constants γ_i and κ_i are

defined, in this case, the values used were found by Chen et al. [37], and they are optimized for combined loads at the tip of the beam.

The program then checks if the angle of any of the force loads is higher than π , if true, the angle is converted from a positive angle higher than π to a negative angle lower than $-\pi$. This is done to prevent some errors found in the solution when using angles with a value higher than π .

Afterwards, the program initiates a *for* loop using *ramp* as the index variable, this variable is used to slowly increase the value of the loads up to their respective values. This is done to ensure that the solver *fsolve* is able to calculate the correct solution by taking the result from the previous iteration and using it as the starting point for the next iteration. Inside this *for* loop, a different *for* loop is used to iterate using the variable *j* to create the statics equations for each beam, by calling the function *statics*.

The function *statics* uses equations (3-7) to (3-13) to create the moment functions for each torsion spring. A general explanation of how it works is given in Figure 3-9.

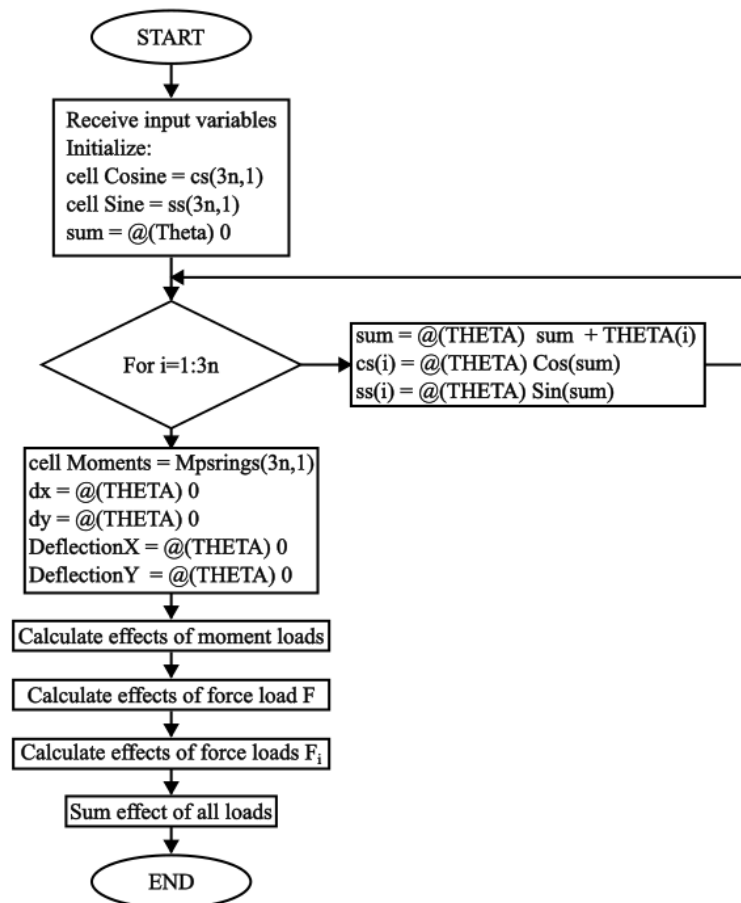


Figure 3-9.- Flow diagram for Statics function to create the spring moment system of equations.

The *statics* function receives the input variables from the main program, as well as the current values for the angle of each subsegment of each beam Θ_i , this is done to calculate the sine and cosine coefficients from equations (3-9), (3-11) and (3-12).

Since *fsolve* requires a system of equations that includes all of the moments for each spring and since this problem is non-linear, typical arrays cannot be used to create the system of equations. A special type of array called cell array is needed. The advantage of cell arrays is that they can store different types of data, including anonymous functions. In MATLAB, these are the functions starting with @(x), where x is an unknown variable that needs to be solved, and it is the type of input that *fsolve* requires.

After exiting the for loop with the index variable i , the program initializes the variables required to calculate the deflection displacements defined by equation (3-12). Finally, in the final 4 blocks, the program uses equations (3-7) to (3-13) to create the moment equation of each torsion spring.

Function *CreateFun* from Figure 3-8 is much simpler as it only converts the output of *statics*, which is a cell array containing all spring moment functions, into a non-cell type of array as *fsolve* cannot work with cell arrays.

Finally, function *Positions* takes the spring angles $\Theta_1, \Theta_2, \dots, \Theta_{3n}$ calculated in the main program using *fsolve* and plots the position of each spring based on Θ_i , γ_i and l_i . A visual comparison between ANSYS Workbench (Figure 3-10.a), COMSOL (Figure 3-10.b) and the MATLAB (Figure 3-10.c) results is given in Figure 3-10.

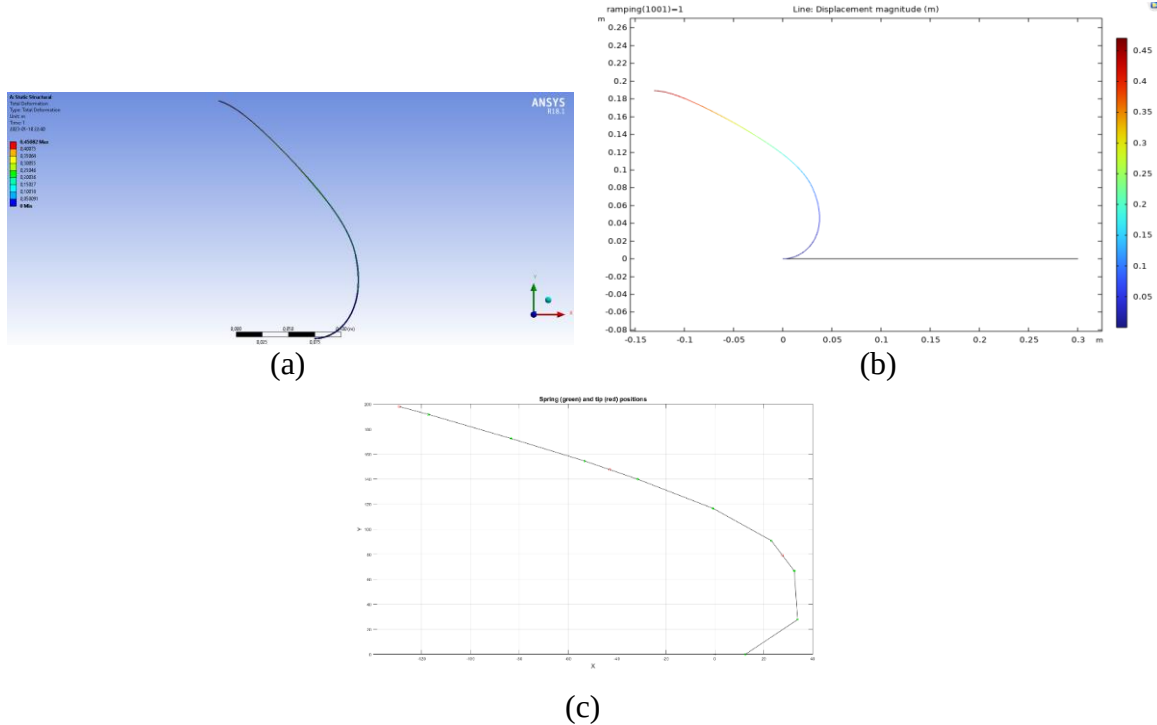


Figure 3-10.- Comparison of three cantilever beams connected in series by rigid joints, when subjected to a combined end load of a 10 N force at 150° and 0.01 Nm CCW moment. Solution in (a) ANSYS Workbench, (b) COMSOL and (c) MATLAB.

A quantitative comparison of the results between the PRBM and FEM is shown in Table 3-4, this compares the deflection with ANSYS and COMSOL for 16 load cases, which range from force loads of 0.0005 N to 0.001 N with varying angles, while also incorporating moment loads ranging from 0.000025 Nm to 0.0025 Nm. The loads are relatively small due to the low thickness of the beam model.

Thickness	Force (N,degrees)	Moment (Nm)	Average error for each tip position					Average Error	
			%e MATLAB vs APDL	%e MATLAB vs WBs	%e MATLAB vs WBd	%e MATLAB vs COMSOLs	%e MATLAB vs COMSOLd	Static	Dynamic
0.16mm	5e-4,90	2.5e-4 CCW	0.23	0.08	0.21	0.07	0.09	0.13	0.15
0.16mm	5e-4,45	2.5e-4 CCW	2.82	3.32	2.71	2.82	2.82	2.99	2.77
0.16mm	5e-4,225	2.5e-4 CCW	5.03	5.11	5.28	5.11	5.11	5.09	5.19
0.16mm	5e-3,90	2.5e-4 CCW	0.16	0.15	0.21	0.15	0.15	0.15	0.18
0.16mm	5e-3,45	2.5e-4 CCW	8.37	8.38	8.90	8.41	8.63	8.39	8.77
0.16mm	5e-3,225	2.5e-4 CCW	0.38	0.37	3.63	0.38	0.90	0.38	2.27
0.16mm	9e-4,90	2.5e-4 CCW	0.39	0.29	0.18	0.08	0.08	0.25	0.13
0.16mm	9e-4,45	2.5e-4 CCW	4.23	4.23	4.33	4.23	4.23	4.23	4.28
0.16mm	9e-4,225	2.5e-4 CCW	13.99	13.98	13.82	13.98	13.98	13.98	13.90
0.16mm	9e-4,90	2.5e-5 CCW	0.11	0.11	0.11	0.11	0.11	0.11	0.11
0.16mm	9e-4,45	2.5e-5 CCW	8.60	8.93	8.69	8.59	8.59	8.71	8.64
0.16mm	9e-4,225	2.5e-5 CCW	8.20	8.57	8.09	8.20	8.20	8.32	8.14
0.16mm	5e-4,90	2.5e-5 CCW	0.11	0.10	0.18	0.10	0.10	0.10	0.14
0.16mm	5e-4,45	2.5e-5 CCW	7.49	7.74	7.44	7.49	7.49	7.57	7.47
0.16mm	5e-4,225	2.5e-5 CCW	13.47	11.37	11.25	11.37	11.37	12.07	11.31
0.16mm	1e-3,135	2.5e-3 CCW	0.23	0.22	0.12	0.25	0.25	0.23	0.19
								4.54	4.60

Table 3-4.- MATLAB results for the PRBM compared to FEM results using ANSYS Workbench static (WBs) and dynamic (WBd), COMSOL static (COMSOLs) and dynamic (COMSOLd).

This comparison was made for the same geometry but different load types, varying the force, its angle and the moment, the force is on the tip of the third beam and the base of the first beam is connected to a fixed support. A static solution is given in ANSYS Workbench and COMSOL, along with a dynamic solution using ANSYS Workbench and COMSOL. Since the PRBM is based on static equations it is necessary to apply a quasistatic load on the dynamic model to properly compare results, not doing this will lead to a different solution as inertial terms start to become more important. The error is calculated by the position of the tip of each beam and an average error is given for each solver. Finally, an average error is calculated for the static and dynamic models when compared to MATLAB results. The error is calculated by comparing the displacement of the tip of each beam using equation (3-14). Then, the average error is shown in Table 3-4 as the average of the three beam's tips displacement errors. Finally, an average calculation of the error of all static and dynamic errors is given in the bottom right corner of Table 3-4.

$$\%e_{tip\ displacement} = \frac{|T_{d_{MATLAB}} - T_{d_{FEM}}|}{T_{d_{FEM}}} \quad (3-14)$$

Where:

$T_{d_{MATLAB}}$: Total displacement calculated by MATLAB.

$T_{d_{FEM}}$: Total displacement calculated by FEM software.

$\%e_{tip\ displacement}$: Percentage error of tip displacement.

The average error of the static models is 4.54% and for the dynamic models is 4.60%, each error is quite acceptable. This implementation of equations (3-7) to (3-13) also

works when the dimensions of each beam are different, as well as their elastic modulus. Table 3-5 shows these results, where the thickness of the three beams in series ranges from 0.16 to 1 mm, while at the same time, the elastic modulus varies from 41 to 60 GPa.

Thickness (A,B,C)	Young's Modulus (A,B,C)	Load (Force, Angle, Moment)	Average error for spring positions				Average Error	
			%e MATLAB vs WBs	%e MATLAB vs WBd	%e MATLAB vs COMSOLs	%e MATLAB vs COMSOLd	Static	Dynamic
0.16 mm, 0.3 mm, 1 mm	41 GPa, 50 GPa, 60 GPa	1e-3 N, 135°, 2.5e-3	0.56	0.56	0.58	0.58	0.57	0.57
0.8 mm, 0.5 mm, 0.16 mm	60 GPa, 50 GPa, 41 GPa	1e-3 N, 135°, 2.5e-3	0.82	0.82	0.83	0.83	0.82	0.83
1 mm, 0.3 mm, 1 mm	41GPa, 50 GPa, 41 GPa	1e-3 N, 135°, 2.5e-3	1.38	1.40	1.38	1.38	1.38	1.39
							0.92	0.93

Table 3-5.- MATLAB results for the PRBM compared to FEM analysis using different geometries and Young's modulus for each beam.

For this test, a single load case is used, and the thickness of each beam varies along with their Young's modulus. A static solution is given for each problem using ANSYS Workbench and COMSOL, and the results are compared with the PRBM MATLAB results. The average error is calculated using equation (3-14), which are quite small below 1%.

As shown in Table 3-4 and Table 3-5, the MATLAB implementation of the pseudo-rigid body model with three revolute joints shows accurate results when compared with FEM analysis software, such as ANSYS Workbench and COMSOL. The entire purpose of this implementation is to incorporate optimization algorithms into the code and to be able to calculate the required displacement of a SMA actuator in order to generate a grasping force for a gripper design.

Before doing this, however, the SMA material should be analyzed in order to obtain its material properties, since the PRBM relies on this information to perform the large deflection analysis.

4. NUMERICAL SIMULATION OF A CANTILEVER SHAPE MEMORY ALLOY SHEET BASED ON EXPERIMENTAL DATA.

4.1. MATERIAL SPECIFICATIONS.

In this chapter, the experimental and numerical testing will be conducted using SMA materials in order to quantify their mechanical properties and their performance characteristics.

The material was purchased from Nexmetal Corporation, it consists of a 1 mm thick SMA sheet with dimensions 10.2 cm by 20 cm. Nexmetal Corporation provides the austenite finish transformation temperature (A_f) and it is specified as $60\text{ }^{\circ}\text{C} \pm 10^{\circ}\text{C}$. Apart from that, no more information is provided. This is one of the challenges of using prefabricated materials, as there is no control over their mechanical properties unless the designer is able to fabricate their own materials. In this methodology, it is necessary to quantify the mechanical properties of the material in order to perform various calculations required for the design of the gripper actuators.

Two rectangular shaped SMAs were cut with the objective of using them as actuators of a SMA-based gripper. The SMAs are Ni-Ti alloys, which are reflective, this means that they are able to reflect photons, and they are difficult to cut, while also being sensitive to temperature, since it could affect their material properties and behavior, making them not suitable for being cut using methods such as laser or plasma cutting.

The amount of material is also restricted, this limits the possibility of using a milling CNC machine, since this method for cutting the actuators would lead to some material waste while the cut is being made.

Cutting the material with an abrasive disk generates less waste and a constant stream of water can help mitigate the rise in temperature of the material, lowering the effects of temperature on the material properties of the actuators. It is still possible that the temperature will affect the material properties, however, since less material is wasted, more samples can be cut from the main source.

The SMA-based gripper requires two sheets, and for this purpose, two samples were cut using an abrasive disk. The alloy is 1 mm thick, and the rectangular samples were cut to have a length of 120 mm and a width of ~10 mm.

4.2. SENSITIVITY ANALYSIS.

In order to better understand the behavior of a shape memory alloy material, consider the cantilever beam subjected to a force that increases from 0 to 10 N, as shown in Figure 4-1. The force will induce a displacement on the beam depending on its geometric dimensions and material properties. It is easy to measure the geometric dimensions, however, the material properties must be calculated based on experimental data.

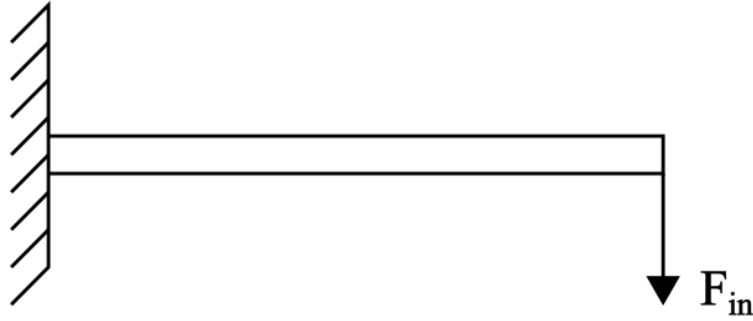


Figure 4-1.- Cantilever beam subject to a vertical load.

Ideally, the material should be subject to a tension test, however, this type of test requires a specific sample shape, meaning that a large part of the material would be used for mechanical properties calculation. Also, tests should be done in controlled environments where the temperature remains constant, in order to have accurate data. This makes it difficult to carry out, thus, a simpler approach is proposed: The aforementioned cantilever beam is subject to a load that increases gradually from 0 to 10 N. This will induce a deflection on the beam, which depends on the material properties of the SMA. This experimental setup is then introduced to a numerical simulation, for example, using ANSYS Workbench, and the numerical test is done in various steps from 0 to 10 N. Then the material properties of the SMA that better match the experimental data can be estimated in order to represent the behavior of the beam in the numerical simulation.

An initial analytical analysis can be performed, however, there are variables that alter the effects of the results, depending on the temperature of the beam. Based on the cantilever beam shown in Figure 4-1, the displacement of the beam can be calculated using the well-known formula for the displacement of a cantilever beam with an end load, shown as equation (4-1).

$$y_f = -\frac{Fl^3}{3EI} \quad (4-1)$$

The SMA numerical model available in ANSYS requires the material properties shown in Table 4-1.

Property	Symbol
Young's modulus (Martensite) (MPa)	E_m
Young's modulus (Austenite) (MPa)	E_a
Poisson's ratio	ν
Tangent modulus (MPa)	E_t
Elastic limit (MPa)	S_y
Temperature scaling parameter (MPa $^{\circ}\text{C}^{-1}$)	TSP
Maximum transformation strain (mm mm^{-1})	MTS
Loade dependency parameter	α

Table 4-1.- ANSYS shape memory alloy required material properties.

As shown in Table 4-1, the numerical simulation model requires various material properties not considered by equation (4-1). To estimate the effect of the material properties shown in Table 4-1, a sensitivity analysis will be performed. This analysis will make it easier to better understand how each parameter affects the displacement results of a cantilever beam subject to an end load. This is required in order to accurately measure the mechanical properties of the available materials and, since it is difficult to measure some of these values without proper equipment, it will help in the process of proposing a probable range of some unknown parameters that accurately represent the experimental behavior of the material. Various numerical simulations were run where the base material properties are extracted from [43], and shown in Table 4-2.

Property	Symbol	Value
Young's modulus (Martensite) (MPa)	E_m	19260
Young's modulus (Austenite) (MPa)	E_a	45440
Poisson's ratio	ν	0.3
Tangent modulus (MPa)	E_t	659.93
Elastic limit (MPa)	S_y	110.19
Temperature scaling parameter (MPa $^{\circ}\text{C}^{-1}$)	TSP	1.43
Maximum transformation strain (mm mm^{-1})	MTS	0.061
Lode dependency parameter	α	0

Table 4-2.- Base material properties of the SMA material model [43].

These base values are used as a reference to compare the displacement results of different numerical simulations where some of these values are increased and decreased by 25%. For this purpose, two types of analysis have to be conducted, one in a low temperature setting, that represents the martensite state of the material, and one in a high temperature setting, where the material is in the austenite phase. Two base analysis results are shown in Figure 4-2. These two graphs show the behavior of the beam model using the material properties shown in Table 4-2.

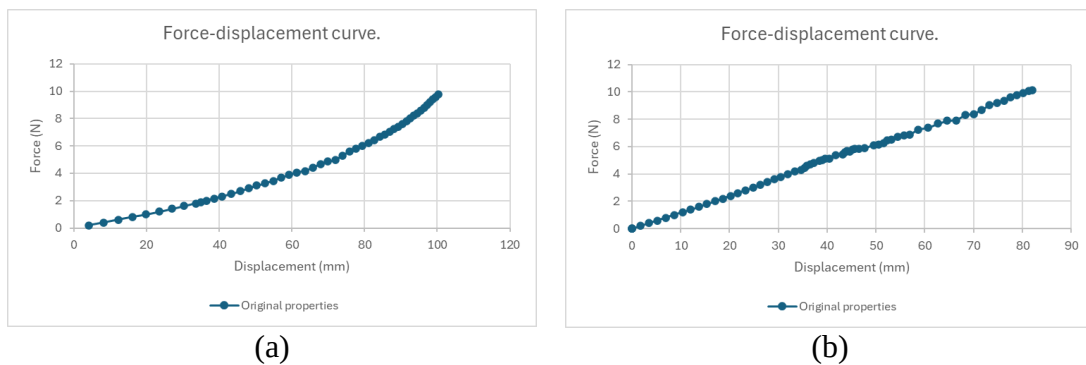


Figure 4-2.- Numerical force-displacement curve in (a) martensite phase and (b) austenite phase using base material properties.

The amount of data points in these curves are too many to include here. Four points are chosen as a reference when comparing results in the following pages. For simplicity, the

four points that are used as a reference are 2.5 N, 5 N, 7.5 N and 10 N, the reference results are shown in Table 4-3.

Force (N)	Displacement in martensite phase (mm)	Displacement in austenite phase (mm)
2.5	43.29	21.75
5	73.97	39.05
7.5	90.45	62.68
10	101.27	81.97

Table 4-3.- Reference points for results comparison.

These reference points will be useful to compare the material behavior when varying different material properties.

Equation (4-1) can be used to calculate the displacement of the beam, however, the results differ from those shown in Table 4-3. This is probably because equation (4-1) does not consider large deflections, making the resulting displacement much larger than it should be, such is the case of the martensite phase displacement when the load is 10 N, as shown in Table 4-4.

Force (N)	Displacement in martensite phase (mm)	Displacement in austenite phase (mm)
2.5	51.94	22.01
5	103.88	44.03
7.5	155.82	66.04
10	207.76	88.06

Table 4-4.- Reference points displacements calculated using equation (4-1).

Since the analytical equation (4-1) does not consider large deflections, it is better to use FEA to understand the effects of the various material properties shown in Table 4-2. Furthermore, the numerical simulations can be reused to estimate the actual properties of the SMA material based on experimental results, this process is performed in section 4.3.

4.2.1. MARTENSITE PHASE SENSITIVITY ANALYSIS.

The elastic modulus has a very important role in the displacement of a cantilever beam when subject to an end load. This is true in both temperature settings, since it dictates how much the beam will bend.

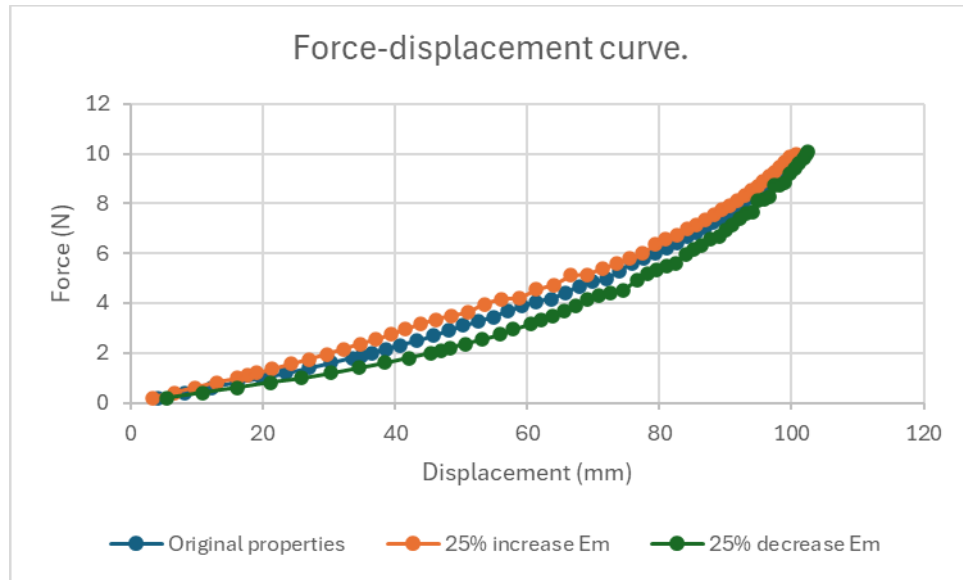


Figure 4-3.- Martensite phase elastic modulus E_m sensitivity analysis.

As shown in Figure 4-3, the behavior mostly changes in the middle of the curve, and a 25% decrease in E_m seems to have a higher impact than a 25% increase in E_m . This is easier to understand using the actual results when varying E_m and comparing them with the reference points.

Force (N)	Displacement using original values (mm)	Displacement with a 25% increase in E_m (mm)	Displacement with a 25% decrease in E_m (mm)
2.5	43.29	37.05	53.16
5	73.97	66.59	78.12
7.5	90.45	88.28	93.05
10	101.27	100.60	102.47

Table 4-5.- Reference points comparison when varying the martensite phase elastic modulus E_m .

As shown in Table 4-5, the results vary the most in the lower force range. The higher force range is not affected as much, probably due to the elastic limit, especially when the force is 10 N. At this point, it is likely that most of the displacement is due to a fully plastic cross section, where the elastic modulus has been replaced by the tangent modulus. It would require a dramatic increase in the elastic modulus for the beam to not displace as much with this force. Also, a higher elastic limit would probably extend the region where the curves diverge, at the moment, these diverge probably close to where the elastic limit is reached and converge afterwards due to the effect of the tangent modulus.

When using a force of 2.5 N, a 25% increase in the elastic modulus produces up to a 16.7% decrease in displacement, opposite to this, a 25% decrease in the elastic modulus produces up to an 18.5% increase in displacement. This is logical since increasing or decreasing the stiffness of the material will produce a contrary effect on how much the

material displaces with the same force. The more the force increases, the lower the impact of modifying the material properties.

The second most important material property in martensite phase analysis is the elastic limit. The elastic modulus dictates how much the material will bend, while the elastic limit dictates when it will start to bend. Figure 4-4 shows the force-displacement curves when varying the elastic limit material property.

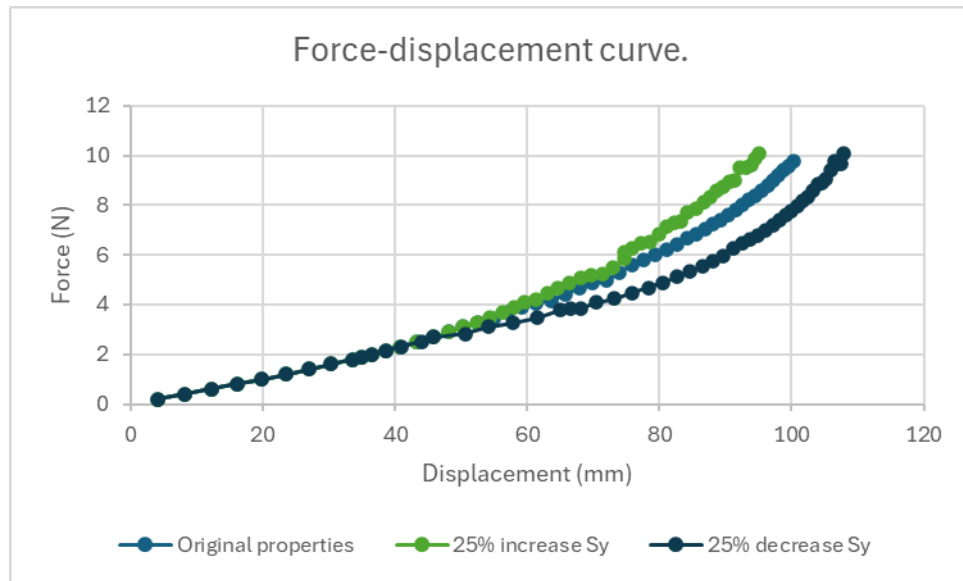


Figure 4-4.- Martensite phase elastic limit S_y sensitivity analysis.

As opposed to the elastic modulus, varying the elastic limit has a more prominent effect with higher force ranges. This is expected since it is directly tied to the stress applied to the beam while bending due to the action of the force and it does not have an effect until after the stress is equal to the elastic limit. The comparison with the reference results is shown in Table 4-6.

Force (N)	Displacement using original values (mm)	Displacement with a 25% increase in S_y (mm)	Displacement with a 25% decrease in S_y (mm)
2.5	43.29	43.29	44.10
5	73.97	68.08	82.69
7.5	90.45	84.27	99.09
10	101.27	95.06	107.91

Table 4-6.- Reference points comparison when varying the elastic limit S_y in the martensite phase.

As shown in Table 4-6, the effect is most noticeable after the 2.5 N force reference point. It is also important to note that decreasing the elastic limit by 25% does have a small impact on the result at 2.5 N of force, probably due to the elastic limit being low enough

for a relatively small force to start the yield of the material. The results vary by up to 10% when decreasing the elastic limit and using a force of 5 N.

In the martensite phase only the martensite modulus and the elastic limit have an important impact on the behavior of the beam. The tangent modulus varies the results by less than 1% in all reference points, while the temperature scaling parameter and the maximum transformation strain have zero impact on the results.

4.2.2. THE EFFECT OF THE TANGENT MODULUS IN THE FORCE-DISPLACEMENT CURVE OF A PERFECT PLASTIC MATERIAL.

In order to estimate the tangent modulus of the SMA material, it is important to have a good understanding of how the force-displacement graph should behave depending on the tangent modulus after the cross section has transitioned to a fully plastic behavior, this is because, as mentioned in section 4.2.1, the tangent modulus does not have an important effect on the final results of the displacement of the beam, however, it does have an effect on the behavior of the force-displacement curve. To exemplify this, consider an ideal elastoplastic material. Roark states in [44] that the stiffness defines the slope of the force-displacement curve up to a force where, based on the geometric properties and the material itself, the resultant moment has transitioned the entire cross section of the beam from an elastic behavior to a plastic behavior, this point corresponds to the fourth stress stage in Figure 4-5.

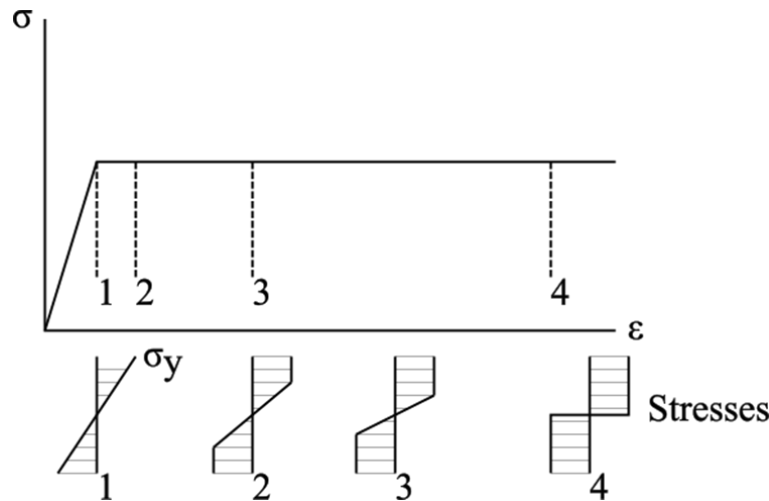


Figure 4-5.- Ideal elastoplastic material stress-strain curve and beam cross section stresses.

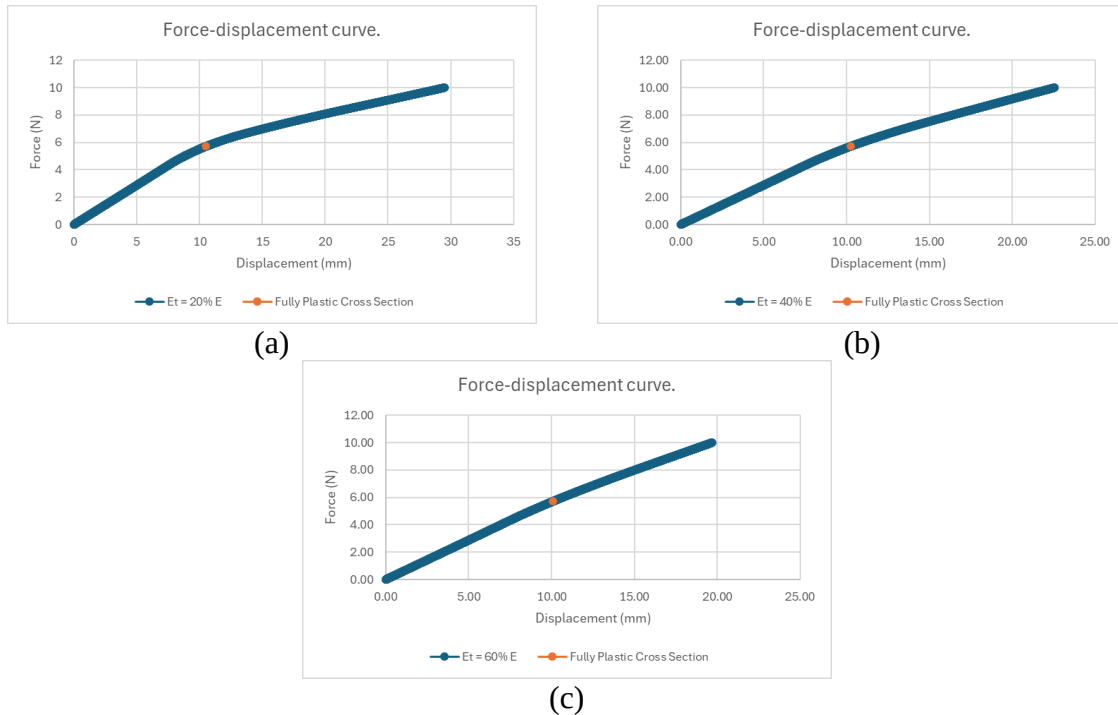
At this point, called the plastic moment M_p , the transition from elastic to plastic behavior is complete and the tangent modulus fully determines the behavior of the material.

In large deflection analysis, the moment is not calculated by simply multiplying the length of the beam with the applied end load, since the shortening of the lever-arm has to be considered. This is not a problem in FEM since the moment can be calculated at any sub-step, however, it should be considered when performing experimental testing. In this case, the plastic moment M_p is calculated using equation (4-2).

$$M_p = \frac{1}{4}(\sigma_y b h^2) \quad (4-2)$$

Where σ_y represents the yield strength of the material, b and h represent the base and height of the rectangular cross section, respectively. Suppose the elastoplastic material has an elastic modulus E of 200 GPa and a yield strength of 200 MPa, in addition, its geometry is a simple beam with a length $L = 100$ mm and a rectangular cross section with a base $b = 11.35$ mm and a height $h = 1$ mm. The beam is loaded as a cantilever beam with a simple end load that increases from 0 to 10 N, as shown in Figure 4-1.

For this example, the plastic moment M_p is 567.5 Nmm, this value is reached when the force at the tip of the beam is equal to 5.71 N for this particular case. After the force reaches 5.71 N, the behavior is completely dependent on the tangent modulus. To analyze the effects of this material property, consider three cases, where the tangent modulus is equal to 20% (Figure 4-6.a), 40% (Figure 4-6.b) and 60% (Figure 4-6.c) of the elastic modulus. These values should provide a good understanding of how the force-displacement graph should behave depending on the tangent modulus after the cross section has transitioned to a fully plastic behavior.



**Figure 4-6.- Elastoplastic material behavior depending on the tangent modulus E_t .
Where (a) $E_t = 20\% E$, (b) $E_t = 40\% E$, (c) $E_t = 60\% E$.**

As shown in Figure 4-6, as the tangent modulus increases, the difference between the slope of the elastic behavior of the beam and the slope of the plastic behavior of the beam decreases. The slopes are calculated for both the elastic and plastic zones, considered to

the left and right of the orange point, respectively. The orange point represents the required force to reach the plastic moment M_p for each of the displacement graphs. The percentage difference is shown in Table 4-7.

Case	Elastic slope	Plastic slope	Percentage difference
$E_t = 20\% E$	0.56	0.22	61.11
$E_t = 40\% E$	0.57	0.34	39.69
$E_t = 60\% E$	0.57	0.44	22.59

Table 4-7.- Percentage difference of elastic and plastic zone sloopes for each elastoplastic material case.

There is no clear mathematical relation between the specific values of the tangent modulus of each case and the corresponding slopes. However, this behavior should be consistent regardless of the material. With this information, by analyzing the experimental force-displacement curve of the SMA material, it may be possible to estimate the tangent modulus by comparing the force-displacement curve with numerical simulation results. Modifying the tangent modulus to be 60%, 40% or 20% of the martensite phase elastic modulus E_m will affect the slope of the material after it has reached the elastic limit. Therefore, it is crucial to have an experimental setup that encompasses both the elastic and plastic zones of the force-displacement curve.

4.2.3. AUSTENITE PHASE SENSITIVITY ANALYSIS.

As explained before, the most important material property in a cantilever beam analysis is the elastic modulus of the material. As expected, increasing or decreasing the elastic modulus of the material affects the results, similarly to the martensite phase behavior, with a corresponding decrease or increase in the displacement of the tip of the beam. This is shown in Figure 4-7.

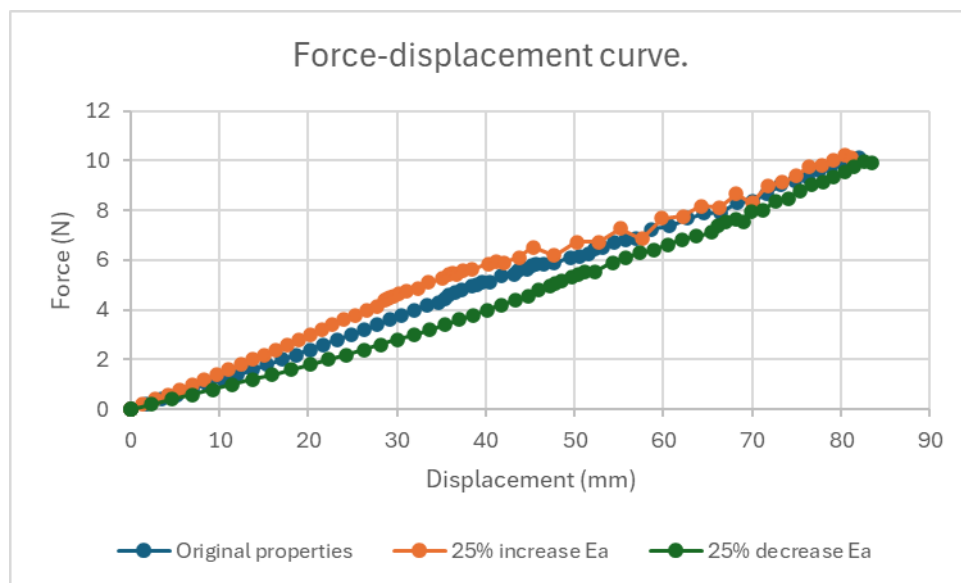


Figure 4-7.- Austenite phase elastic modulus E_a sensitivity analysis.

As expected, altering the elastic limit diverges the results in a relatively low force range, however, as the force increases, the results start to converge once again. The explanation is the same as in the martensite phase, at some point, probably at the elastic limit, the stress increases and the beam yields, mitigating the effect of changing the elastic modulus.

Table 4-8 shows the comparison of the reference values with the displacement values obtained from varying the elastic modulus E_a of the austenite phase.

Force (N)	Displacement using original values (mm)	Displacement with a 25% increase in E_a (mm)	Displacement with a 25% decrease in E_a (mm)
2.5	21.75	17.65	28.17
5	39.05	33.51	47.85
7.5	62.68	59.84	67.01
10	81.97	81.17	83.45

Table 4-8.- Reference points comparison when varying the austenite phase elastic modulus E_a .

The results vary the most with lower force values, for example, a 25% increase in the austenite elastic modulus results in up to a 23.22% decrease in the total tip displacement of the cantilever beam. Likewise, a 25% increase in the austenite phase elastic modulus results in up to a 22.79% decrease in the total displacement. This variation decreases as the force increases.

Once again, similarly to the martensite analysis results, the elastic limit S_y has an impact on the behavior of the cantilever beam when the force is enough to have the necessary stress to yield the material, the results are shown in Figure 4-8.

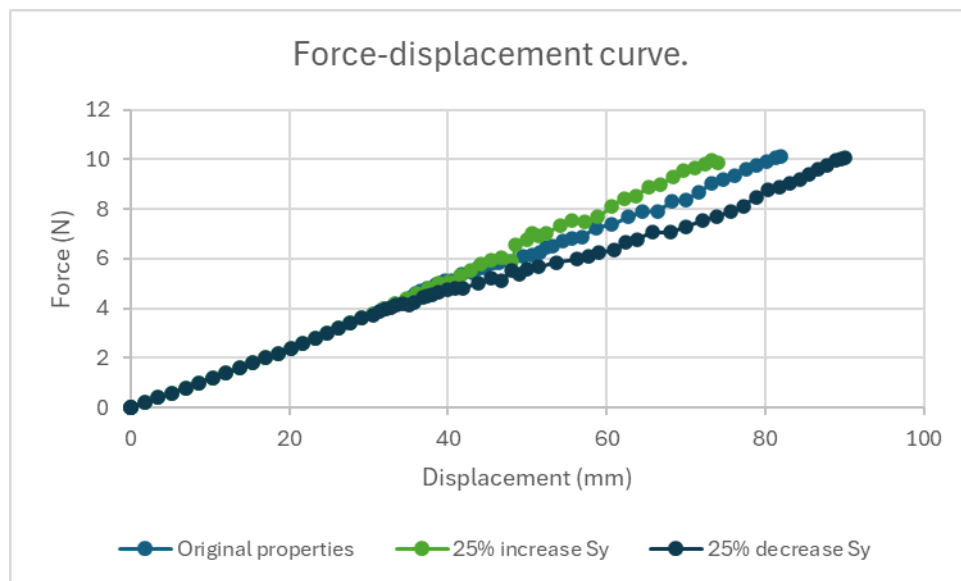


Figure 4-8.- Austenite phase elastic limit S_y sensitivity analysis.

The beam shows the same behavior as in the martensite phase, the comparison with the reference values is shown in Table 4-9.

Force (N)	Displacement using original values (mm)	Displacement with a 25% increase in S_y (mm)	Displacement with a 25% decrease in S_y (mm)
2.5	21.75	21.75	21.75
5	39.05	40.17	43.78
7.5	62.68	55.65	72.05
10	81.97	74.08	90.11

Table 4-9.- Reference points comparison when varying the elastic limit S_y in the austenite phase.

This time, the effects are most noticeable after 5 N, and the results vary by up to 13% when decreasing the elastic limit by 25% when the force is 7.5 N.

In the austenite phase the tangent modulus does not have an important impact on the results either. A possible reason for this could be that the parameter is too small compared to other values, this means that a 25% variation is still relatively small.

The maximum transformation strain also shows little impact on the results, this could be more related to the effect of the material property, since it is more related to the behavior of the material, when going from a low temperature to a high temperature while also being subject to a force. These are not the conditions that are being simulated, instead, the beam increases in temperature and after that, the load is applied to the austenitic beam. On the other hand, the temperature scaling parameter does have an effect on the behavior of the beam, this is expected since it is related to the temperature change of the beam and the increase in yield stress due to the change in temperature. For this reason, it has a similar effect to the elastic limit variable, as shown in Figure 4-9.

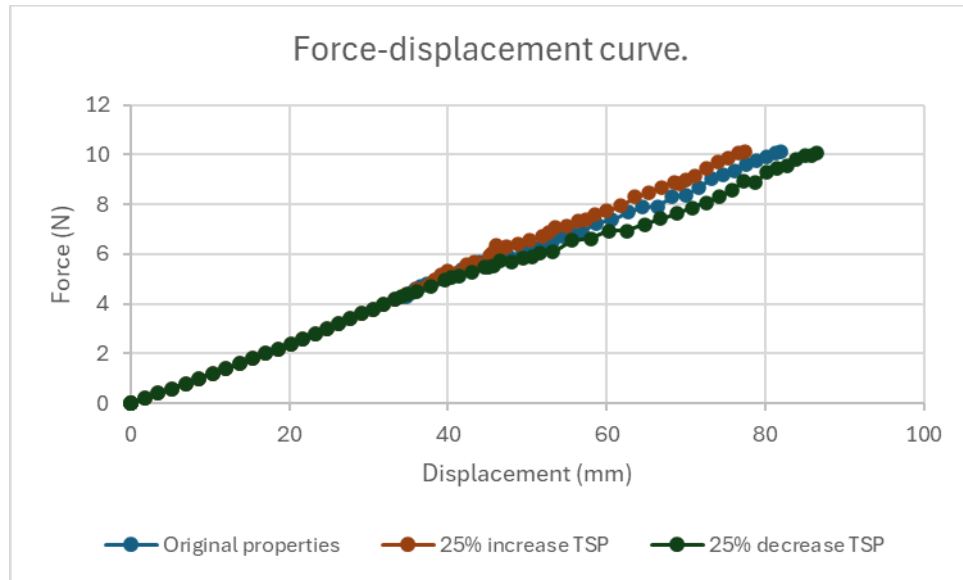


Figure 4-9.- Austenite phase temperature scaling parameter TSP sensitivity analysis.

The behavior is similar to varying the elastic limit, the only difference is that the effect is less intense. The results comparison is shown in Table 4-10.

Force (N)	Displacement using original values (mm)	Displacement with a 25% increase in TSP (mm)	Displacement with a 25% decrease in TSP (mm)
2.5	21.75	21.75	21.75
5	39.05	39.08	40.46
7.5	62.68	58.48	68.84
10	81.97	77.48	86.51

Table 4-10.- Reference points comparison when varying the temperature scaling parameter TSP in the austenite phase.

In comparison to modifying the elastic limit, the maximum variation obtained with the elastic limit was an increase of 13% in displacement by decreasing the elastic limit by 25% when the force is 7.5 N. In this case, the maximum variation is also obtained when the force is 7.5 N when the temperature scaling parameter is decreased by 25%, but the displacement only increases by 8.96%, being lower than when varying the elastic limit.

It is important to note that the elastic limit is easier to estimate from a force-displacement curve, since the temperature scaling parameter is directly related to the stress and the temperature of the beam, making it harder to estimate without proper equipment.

4.3. EXPERIMENTAL ANALYSIS.

With the information obtained from the sensitivity analysis, it is possible to estimate the required changes to the numerical simulation material model in order to better fit the experimental testing results.

The experimental setup consists of two shape memory alloy material samples with a width of ~10 mm, a length of 120 mm and a thickness of 1 mm. The samples were cut using an angle grinder with a 1 mm cut-off wheel, the cuts were made using a constant stream of water to prevent the SMA sheet from overheating. Each shape memory alloy is fixed as a cantilever beam using a clamp, as shown in Figure 4-10.a. The free end is also clamped with a metallic ring, which serves as the connection between the free end and the load. The initial position of the beam is marked using a number 2 pencil to be able to measure the displacement after connecting the load, as shown in Figure 4-10.b.

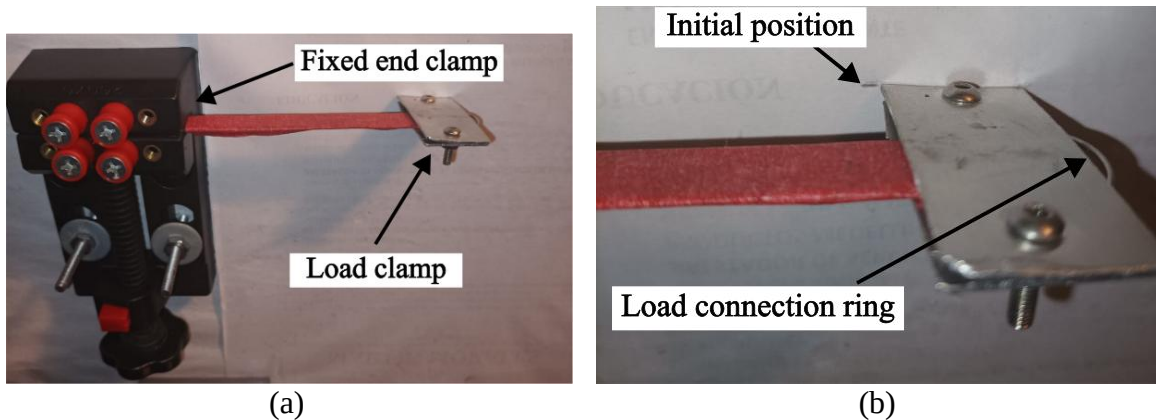


Figure 4-10.- (a) Cantilever beam fixed end and (b) free end experimental setup.

The load ranges from 100 g to 700 g and the displacement is measured every 100 g. The different loads are measured with an ETEKCITY EX6015 digital scale, as shown in Figure 4-11.



Figure 4-11.- Load measurement example.

After the load is applied to the beam, the beam displaces, and the displaced position is marked, as shown in Figure 4-12.

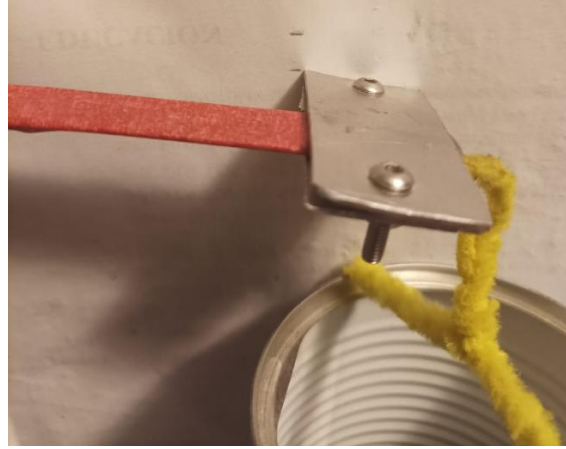


Figure 4-12.- Displaced position mark.

The same process is used for the loads ranging from 100 g to 600 g. The clamp on the free end of the beam has a weight of 6 g, which is added to the loads. Then, the displacement from the initial point to the marks for each load is measured using a digital caliper with a resolution of 0.01 mm and an accuracy of ± 0.02 mm. The force-displacement curve of the beam can be obtained using this process. The SMA beam has a weight of 11 g. The weight of the beam is not considered, the reason can be explained with this example: suppose that the beam has an elastic modulus of $E = 28$ GPa, this is the lower bound for the elastic modulus according to Hodgson [25], using the lower bound, the effects of the load are more important because the beam is less stiff and, in consequence, it deforms more. The beam, as stated before, has a length of $l = 100$ mm and a rectangular cross section of 10 mm by 1 mm, resulting in a cross-section moment of inertia of $I = 0.833 \text{ mm}^4$. The weight of the beam is considered as a distributed load across the entire cross section of $w = 0.11 \frac{\text{g}}{\text{mm}}$. The effect of the end load of 106 g (considering the weight of the clamp) can be calculated using equation (4-1). Using this equation the deflection of the beam due to the end load can be calculated as 14.87 mm.

The effect of the distributed load (the weight of the beam) can be calculated using the formula for the deflection of a cantilever beam due to a distributed load, shown as equation (4-3). Using this equation, the deflection due to the weight of the beam can be calculated as 0.57 mm.

$$y_w = -\frac{w_{dis}l^4}{8EI} \quad (4-3)$$

With the individual effect of the end load and the weight of the beam, the total displacement can be estimated by superposition by adding the results of y_f and y_w , resulting in a total displacement of 15.44 mm. The weight of the beam is not considered because it contributes to only 3.69% of the displacement of the beam for this case, which

will only decrease as the load increases from 106 g to 706 g, while the weight of the beam stays the same.

With the data obtained from the experimental analysis, from the well-known formula of the maximum deflection of a cantilever beam the elastic slope k can be calculated and is used to calculate the elastic modulus by using equation (4-4).

$$E = \frac{kl^3}{3I} \quad (4-4)$$

Where E refers to the elastic modulus of the beam, l is the length of the sample, and I is the moment of inertia of the cross section of the beam.

This experimental setup is ideal for this project since it resembles the bending behavior that the beam will show to be used as a gripper actuator, as a force will induce the opening of the jaws, while the SME will provide the closing motion.

The best course of action is to start by obtaining the martensite phase properties of the samples since, based on the sensitivity analysis from section 4.2.1, it is mostly affected by only three variables, the elastic modulus of the martensite phase, the elastic limit and the tangent modulus E_t .

4.3.1. BEAM-1 MARTENSITE PHASE RESULTS.

The first sample will be referred to as beam-1. Figure 4-13 shows the force-displacement curve of the beam in the martensite phase.

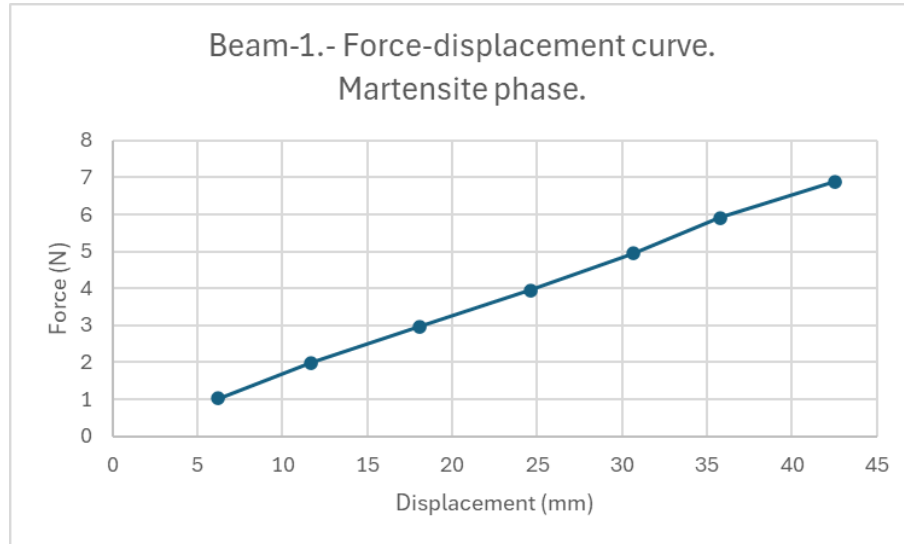


Figure 4-13.- Force-displacement curve for beam-1 in the martensite phase.

Using the data points shown in Figure 4-13, the elastic slope k is calculated, resulting in $k_m = 0.15$. Using equation (4-4), the martensite phase elastic modulus of beam-1 is calculated as $E_{m1} = 54$ GPa.

The elastic limit S_y and the tangent modulus E_t are hard to calculate without the results of a tension test. Hodgson shows some shape memory alloy material properties in [25], from his work, the elastic limit should be in a range from 70 to 140 MPa. With this information, only the tangent modulus is unknown. However, it is known that this material property normally does not exceed the elastic modulus value, with this in mind, various numerical simulations using ANSYS are run, varying S_y in the lower and upper limit, while at the same time varying the tangent modulus, by using 20%, 40% and 60% of the elastic modulus of beam-1.

The various force-displacement curves are compared to the experimental data in Figure 4-14 to obtain an accurate ANSYS model of the material in the martensite phase. In the numerical simulation setup, the force is increased from 0 to 10 N to have a better view of the behavior of the material outside the known experimental setup boundaries.

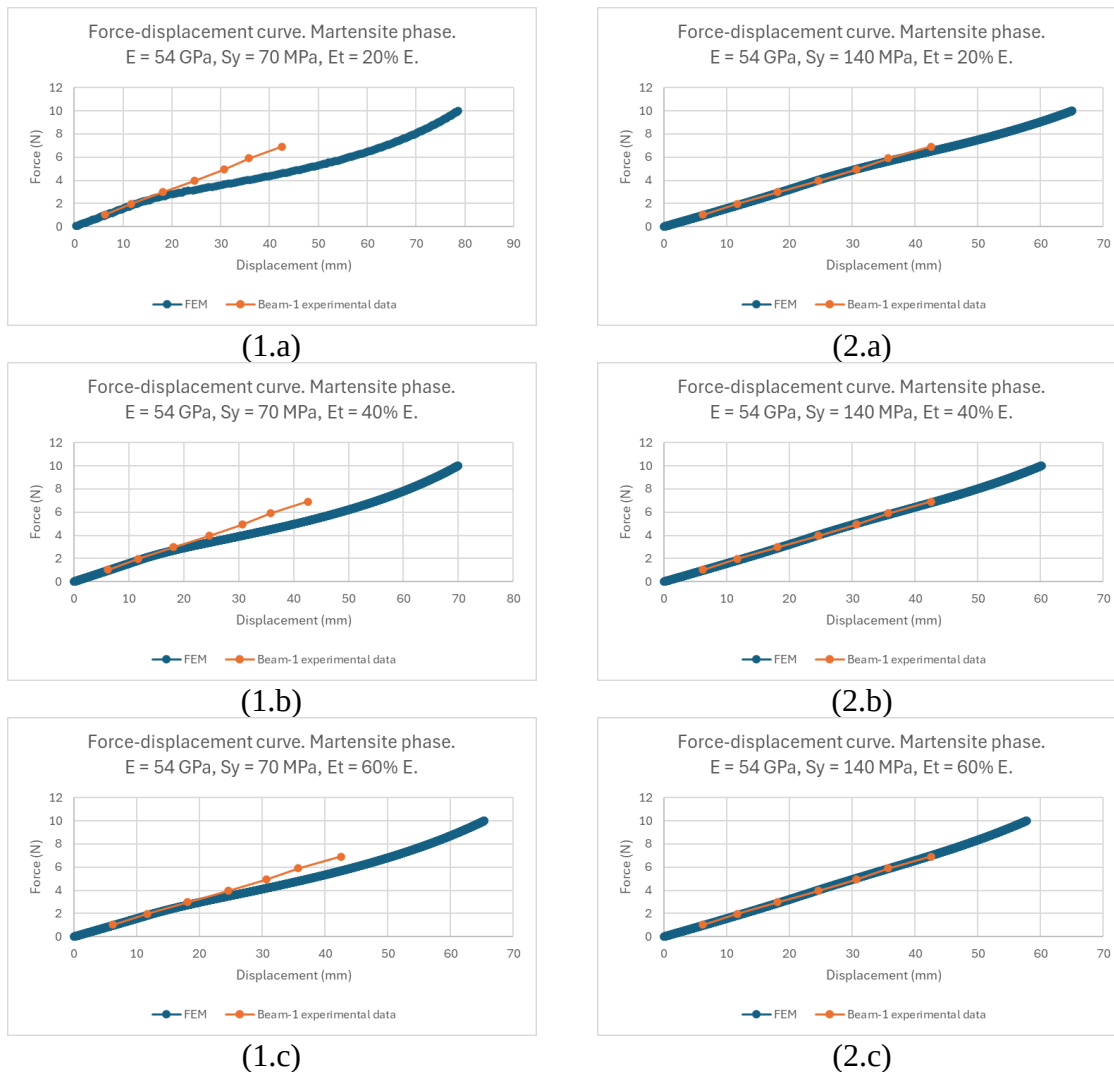


Figure 4-14.- Numerical simulation results for beam-1 in martensite phase, varying S_y and E_t .

Figure 4-14 shows the results using $E_m = 54$ GPa and:

- (1.a) $S_y = 70$ MPa, $E_t = 20\% E_m$, (1.b) $S_y = 70$ MPa, $E_t = 40\% E_m$,
 (1.c) $S_y = 70$ MPa, $E_t = 60\% E_m$, (2.a) $S_y = 140$ MPa, $E_t = 20\% E_m$,
 (2.b) $S_y = 140$ MPa, $E_t = 40\% E_m$, (2.c) $S_y = 140$ MPa, $E_t = 60\% E_m$

In order to measure the precision of the models, a quadratic regression is used to calculate the numerical results based on the available data points from the experimental test, this will lead to an equation similar to equation (4-5).

$$y = ax^2 + bx + c \quad (4-5)$$

The absolute percentage error is calculated by comparing each calculated numerical simulation result with the results obtained during the experimental test, using equation (4-6).

$$e = \left| \frac{x_{\text{experimental}} - x_{\text{simulation}}}{x_{\text{experimental}}} \right| * 100\% \quad (4-6)$$

Model (2.b) and model (2.c) are shown in Figure 4-14, and the calculated error is obtained by comparing each model with the experimental results load cases, estimating the displacement using equation (4-5).

Experimental		FEM Model (2.b)	
Load (N)	d (mm)	d(mm)	e%
1.02	6.25	6.47	3.52
1.99	11.72	12.56	7.17
2.97	18.12	18.60	2.65
3.95	24.60	24.52	0.33
4.94	30.66	30.40	0.85
5.92	35.77	36.10	0.92
6.90	42.56	41.69	2.04
Average error:			2.50

Experimental		FEM Model (2.c)	
Load (N)	d (mm)	d(mm)	e%
1.02	6.25	6.53	4.48
1.99	11.72	12.64	7.85
2.97	18.12	18.57	2.48
3.95	24.60	24.19	1.67
4.94	30.66	30.26	1.30
5.92	35.77	36.65	2.46
6.90	42.56	43.14	1.36
Average error:			3.09

Table 4-11.- Numerical model results comparison to the experimental test results for force-displacement test for beam-1 in the martensite phase.

The results from Table 4-11 show the absolute percentage error for model (2.b), calculated using equation (4-6), where x is the displacement value for each experimental data point, in addition, the average error is also calculated, resulting in a value of 2.5%, showing better results than model (2.c), which has an average error of 3.09%. Using this approach, a reliable numerical simulation model that can accurately represent the behavior of the beam in the martensite phase was found, as an average error of 2.5% is satisfactory for the purposes of this project.

4.3.2. AUSTENITE PHASE TRIGGERING.

Since SMAs have two distinct phases, the model requires information about each phase. Some variables that had no impact on the martensite phase have a more important role in the austenite phase, as seen in section 4.2.

In order to obtain the experimental data, the material temperature has to increase up to 60°C, this way, the material undergoes a phase change from detwinned martensite to austenite. To do this, a temperature analysis was performed.

Some SME triggering methods were introduced in section 2.4. The simplest SME triggering method consists of convection heat transfer by using hot air to increase the temperature of the SMA. This method is useful for testing purposes only and it is only useful to reset the SMA to its original shape before starting any type of tests. This is because the hot air has to be manually directed at the beam using a heating gun, and slight variations of distance from the heat gun to the alloy, number of passes using the heating gun and power setting of the heat gun can alter the results of the SMA testing. Furthermore, the hot air propelled at a high velocity can move the SMA sheet, making it a possible source of noise when taking measurements during testing.

A simple yet effective heating method consists of conduction heat transfer due to the increase in temperature of a Nichrome (Ni-Cr) resistive sheet, a similar method has been successfully used to activate SMA-based torsional hinges using Nichrome coils [19]. These materials are corrosion-resistant and have an electrical resistivity of around $112\mu\Omega\text{ cm}$, in comparison, copper has an electrical resistivity of around $1.678\mu\Omega\text{ cm}$. This results in the possibility of using thin plates instead of thin wires to be able to create enough resistance to generate heat. Because of this, the material can be placed on top of the SMA sheet and connected to a power supply to provide the necessary heat through conduction heat transfer to activate the SME, as shown in Figure 4-15.

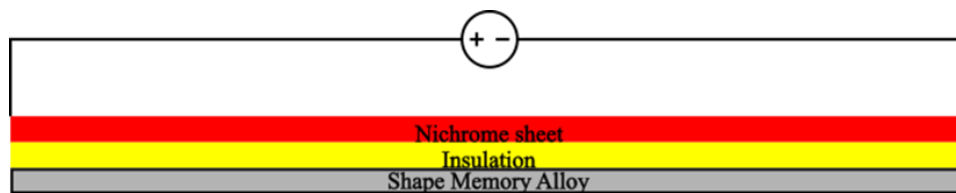


Figure 4-15.- Shape memory alloy and nichrome sheet arrangement for SME activation.

In regard to how to provide the electrical current to the Nichrome resistance element, two methods are selected according to their specific advantages and compared in Table 4-12. The first method is by using a power supply, it has the advantage of delivering power in a controlled manner, in turn, making the temperature rise slower. The second method consists of using a DC battery to deliver as much power as possible to the Nichrome sheet, making the temperature rise much faster. In both cases an insulator interface has to be added between the SMA and the Nichrome sheet, since the SMA is a conductive material.

	Resistance heating with Power Supply	Resistance heating with Battery
Time to activate SME	5 min	5 s
SMA Maximum temperature	65 °C	72 °C
Preparation	-	Battery should charge for 2+ Hours
Power	15.88 W	112.5 W approximately
Repeatability	Heating is slower, the insulator interface does not degrade.	Heating is abrupt, the insulator interface may get damaged if the battery input is not controlled carefully.

Table 4-12.- SMA heating method using Nichrome resistive element.

As shown in Table 4-12, heating using a battery is a faster method, this is because the battery is able to deliver as much power as possible while connected to the Nichrome sheet, that essentially behaves as a short-circuit, in turn, this means that the wiring has to be carefully selected to prevent it from overheating. Similarly, the battery input cannot be driven by most relay devices, instead requiring a high-power relay device.

For application purposes, the shorter response time of the battery method makes it a better alternative, as long as the complications of using a high DC current are considered.

For testing purposes, the power supply method is more reliable, the power is limited by the power supply, preventing overheating, and the power is delivered slowly, avoiding damage to the interface between the SMA and the Nichrome sheet. This is the preferred method for this thesis project and is chosen as the SME triggering method.

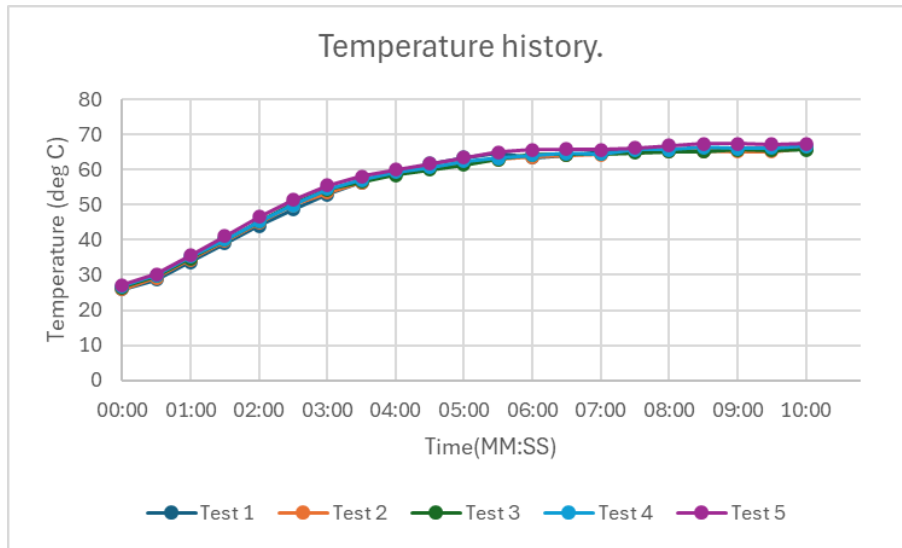


Figure 4-16.- Temperature history using Ni-Cr resistance sheet as heating material.

Figure 4-16 shows the temperature history of beam-1. The temperature sensor is placed opposite to the Ni-Cr resistance in order to capture the temperature of the beam. As shown, the stable temperature is reached after 5 minutes of being connected to the power supply. The average stable temperature after the 5-minute mark is 65.02°C, which is enough to reach the austenite phase.

The weight of the Ni-Cr sheet is not considered since the total weight of the sheet is only 2 g. The experimental setup described in this section is shown in Figure 4-17.

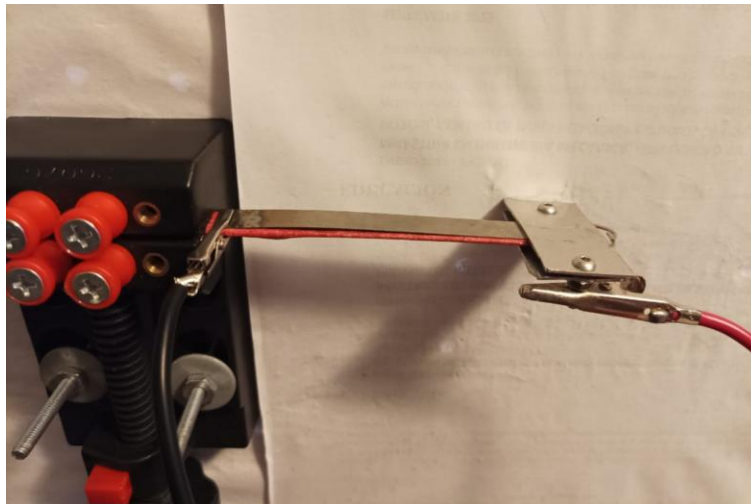


Figure 4-17.- Austenite phase experimental setup.

The Ni-Cr sheet can be clamped using the same clamp used for the load, the weight of the power supply connection should be considered, especially the one on the tip of the beam since the length of the beam causes it to create a moment reaction on the fixed end. For this case, the power supply connection at the tip of the beam adds 5 g to the load.

4.3.3. BEAM-1 AUSTENITE PHASE RESULTS.

After heating the material for 5 minutes, the force load shown in Figure 4-1 is added to the free end of the cantilever beam. The beam is now in the austenite phase. The load is increased from 100 g to 700 g to obtain the data points that will be used to calculate the elastic modulus of the austenite phase of the material using equation (4-4), for this case, the elastic slope k_a is calculated as 0.21 and the resulting elastic modulus is $E_a = 74$ GPa. The force displacement curve is shown in Figure 4-18.

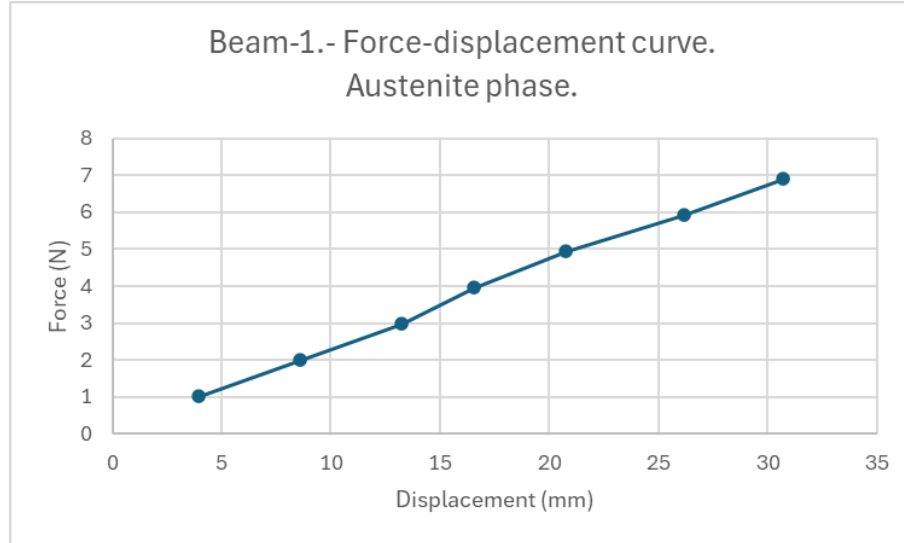


Figure 4-18.- Experimental force-displacement curve for beam-1 in the austenite phase.

Ideally, only the austenite phase elastic modulus should be different from the one used for the martensite phase analysis. With this in mind, the material model is updated to include the austenite phase elastic modulus, and the values obtained for S_y and E_t are kept from section 4.3.1. In the numerical simulation setup, the process is divided into two main steps in ANSYS:

- Step 1: The cantilever beam with no load increases its temperature from 22 to 65 degrees.
- Step 2: The load F increases from 0 to 10 N.

The results of this numerical simulation are shown in Figure 4-19. As shown, the numerical simulation curve heavily resembles the experimental data.

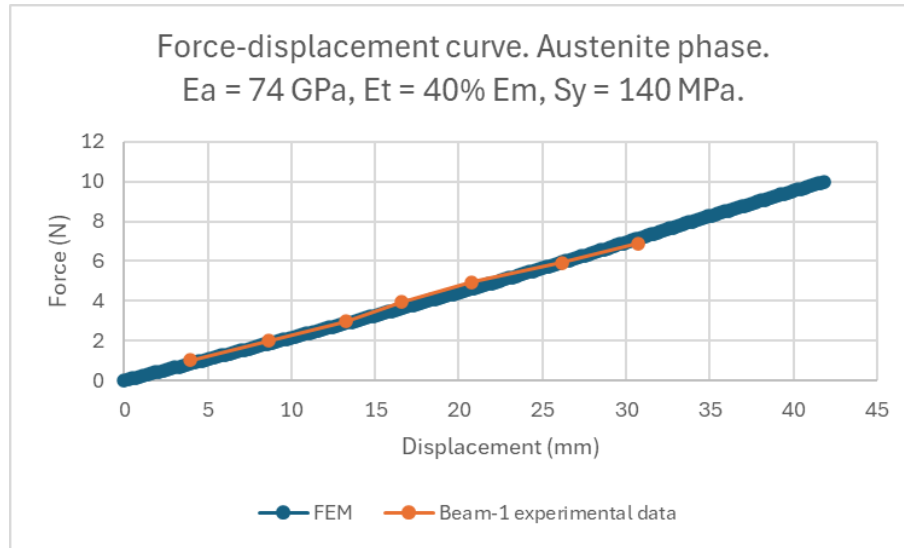


Figure 4-19.- Beam-1 austenite phase numerical simulation results.

The absolute percentage error is calculated using equation (4-6) by comparing the total tip displacement results from the numerical simulation results with the experimental test.

Experimental		FEM model	
Load (N)	d (mm)	d(mm)	e%
1.01	3.96	4.67	17.93
1.99	8.61	9.06	5.23
2.97	13.24	13.81	4.29
3.95	16.57	17.97	8.45
4.93	20.78	22.18	6.75
5.92	26.17	26.20	0.11
6.90	30.73	29.83	2.94
Average error:			6.53

Table 4-13.- Numerical model results comparison to the experimental test results for force-displacement test for beam-1 in the austenite phase.

The results from Table 4-13 show that the material model is able to represent the behavior of beam-1 for the austenite phase. The average error is higher than the average error calculated for the martensite phase results, however, 6.53% is still an acceptable error value for the purpose of this thesis project.

Property	Symbol	Value
Young's modulus (Martensite) (GPa)	E_m	54
Young's modulus (Austenite) (GPa)	E_a	74
Poisson's ratio	ν	0.3
Tangent modulus (GPa)	E_t	22
Elastic limit (MPa)	S_y	140
Temperature scaling parameter (MPa C ⁻¹)	TSP	1.43
Maximum transformation strain (mm mm ⁻¹)	MTS	0.061
Lode dependency parameter	α	0

Table 4-14.- Material properties summary for beam-1.

A summary of the material properties calculated is presented in Table 4-14, note that some of the material properties were not changed from the base material properties from Table 4-2, specifically, the temperature scaling parameter TSP , the maximum transformation strain MTS and the lode dependency parameter α . This is not problematic since these variables do not have an important effect in these loading conditions, as found by the sensitivity analysis from section 4.2.

The obtention of an accurate material model is important in this project because it opens the possibility of simulating the SMA-based gripper to be able to estimate its performance and capabilities.

4.3.4. BEAM-2 NUMERICAL SIMULATION MODEL.

A second sample of the material is taken and will be referred to as beam-2. The same process from section 4.3.1 and 4.3.3 will be followed to corroborate that the methodology followed here is reliable. This sample comes from the same Ni-Ti sheet, however, beam-1 was previously used for other tests and its material properties have likely been altered. Furthermore, this section provides a more concise version of the process followed in those sections.

The force-displacement curve shown in Figure 4-20 is obtained after performing the experiment described in Figure 4-1.

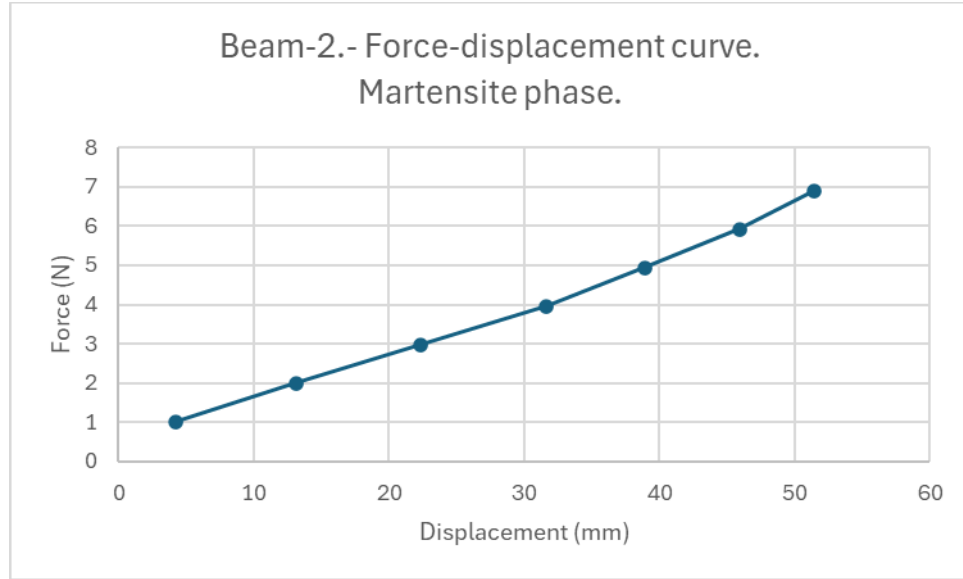


Figure 4-20.- Experimental force-displacement curve for beam-2 in the martensite phase.

Following the same procedure as in section 4.3.1, the first step is to calculate the elastic modulus based on the data shown in Figure 4-20. The elastic slope k_m is calculated as 0.11, then, using equation (4-4), the martensite phase elastic modulus of the beam is calculated as $E_m = 38$ GPa.

The next step is to find the best material model by changing the value of the tangent modulus and the elastic limit. The tangent modulus E_t varies from 40% to 60% of the elastic modulus E_m . The range is reduced due to the results shown in Figure 4-14, where the best results correspond to the material model with a 40% and 60% value of the elastic modulus.

The elastic limit S_y is initially kept at 140 MPa, based on the results of beam-1, also from Figure 4-14. This is because some material properties may remain unchanged even if the elastic modulus has been affected by the various tests that beam-1 has endured. The numerical simulation results are shown in Figure 4-21.

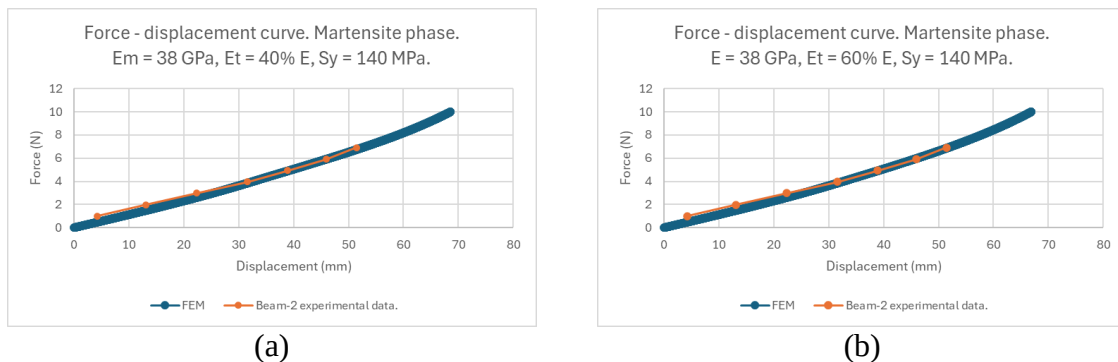


Figure 4-21.- Numerical simulation results for beam-2 in the martensite phase using (a) $E_t = 40\% E_m$ and (b) $E_t = 60\% E_m$.

The numerical simulation results for the model shown in Figure 4-21.b are compared to the experimental test results by calculating the absolute percentage error for each load case by using equation (4-6). This comparison is shown in Table 4-15, where also the average error is calculated.

Experimental		FEM model	
Load (N)	d (mm)	d(mm)	e%
1.01	4.28	8.99	110.05
1.99	13.14	17.40	32.42
2.97	22.35	25.22	12.84
3.95	31.64	32.32	2.15
4.93	38.91	39.06	0.39
5.92	45.94	45.62	0.70
6.90	51.44	51.69	0.49
Average error:			22.72

Table 4-15.- Numerical model results comparison to the experimental test results for force-displacement test for beam-2 in the martensite phase.

As shown in Table 4-15, this numerical model is not as precise as the one found for beam-1. The numerical model average error increases from 2.5% for beam-1 to 22.72% for this model. This decrease in precision may be related to the calculated elastic modulus, since only the first three data points present a very high error relative to the following datapoints, increasing up to 110.05% for the first data point, while the minimum error is 0.39% for the fifth data point. It could be possible to improve the results, however, the results at higher displacements are more relevant in this thesis project, since high displacements are required to obtain detwinned martensite in the martensite phase, for this reason, the obtained results are acceptable.

It is worth noting that the elastic limit value has not changed from beam-1 to beam-2. However, the tangent modulus in beam-1 is 40% of the martensite phase elastic modulus, while for beam-2, it is closer to 60% of the martensite phase modulus. Upon closer examination, considering that the elastic modulus for beam-1 is calculated as $E_{m1} = 54$ GPa, and the elastic modulus for beam-2 is calculated as $E_{m2} = 38$ GPa, the tangent modulus for beam-1 and beam-2 is $E_{t1} = 21$ GPa and $E_{t2} = 22$ GPa. This means that in reality, only the elastic modulus has changed dramatically due to the various tests that beam-1 has been subject to.

For the austenite phase numerical simulation model, using section 4.3.3 as a reference. The first step is to perform the experiment shown in Figure 4-1, ensuring that the beam has reached the required temperature for the phase change, in this case, $A_f = 60^\circ\text{C}$. The experiment results are shown in Figure 4-22.

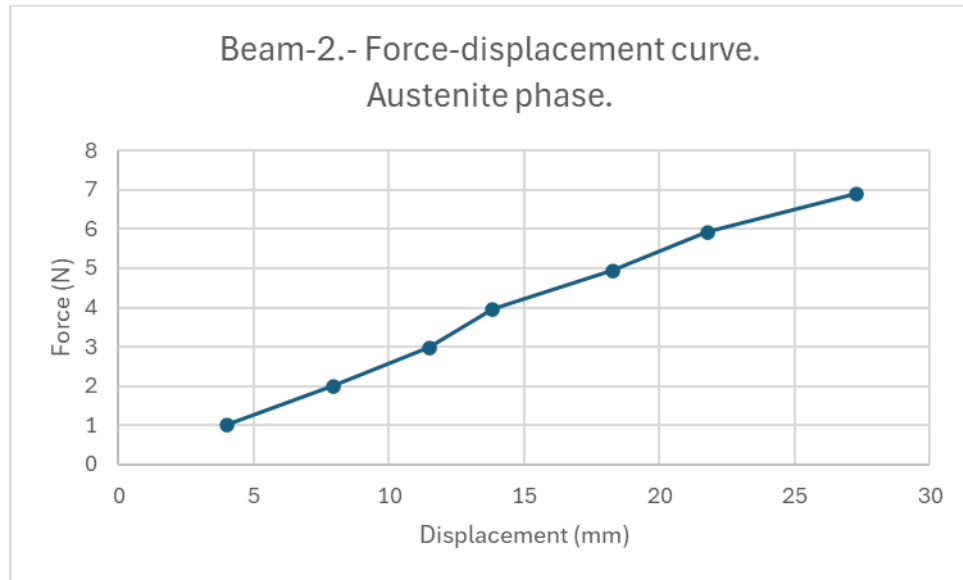


Figure 4-22.- Experimental force-displacement curve for beam-2 in the austenite phase.

Using equation (4-4), the austenite phase elastic modulus is calculated as $E_a = 88$ GPa. Based on the previous assumption, being that only the elastic modulus has changed from beam-1 to beam-2, a single numerical simulation is run, where only the austenite phase elastic modulus is changed for the one of beam-2. The numerical simulation results are shown in Figure 4-23.

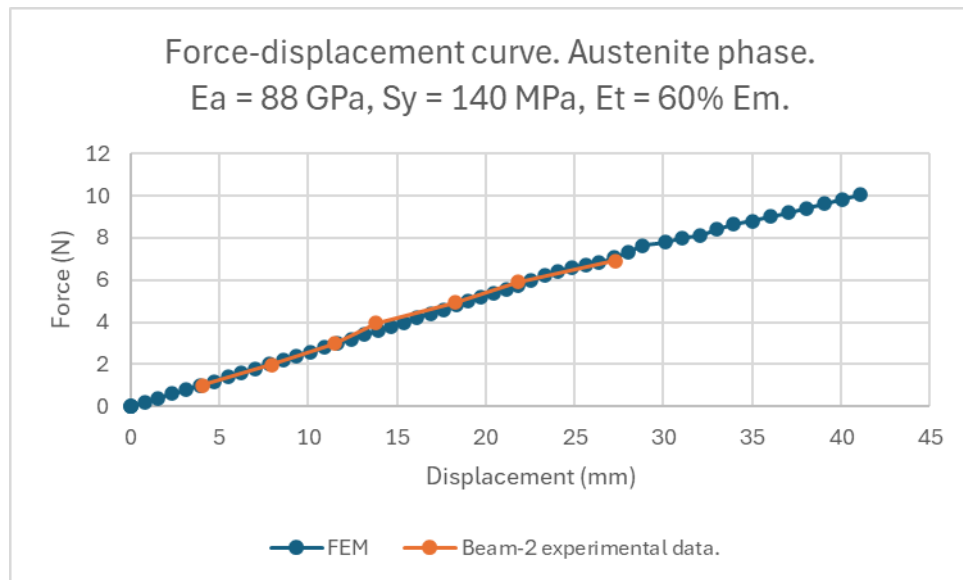


Figure 4-23.- Numerical simulation results for beam-2 in the austenite phase.

Following the established methodology, the experimental force-displacement results are compared to the numerical simulation results.

Experimental		FEM model	
Load (N)	d (mm)	d(mm)	e%
1.01	4.03	3.90	3.23
1.99	7.93	7.79	1.77
2.97	11.48	11.64	1.39
3.95	13.83	14.66	6.01
4.93	18.26	18.31	0.29
5.92	21.78	22.53	3.44
6.90	27.29	27.18	0.40
Average error:			2.36

Table 4-16.- Numerical model results comparison to the experimental test results for force-displacement test for beam-2 in the austenite phase.

The results from Table 4-16 show that this material model accurately represents the experimental test results, since the average error is 2.36%, with a maximum error of 6.01%.

Property	Symbol	Value
Young's modulus (Martensite) (GPa)	E_m	38
Young's modulus (Austenite) (GPa)	E_a	88
Poisson's ratio	ν	0.3
Tangent modulus (GPa)	E_t	22
Elastic limit (MPa)	S_y	140
Temperature scaling parameter (MPa C ⁻¹)	TSP	1.43
Maximum transformation strain (mm mm ⁻¹)	MTS	0.061
Lode dependency parameter	α	0

Table 4-17.- Material properties summary for beam-2.

With the numerical simulation model defined by the material properties shown in Table 4-17, an optimization algorithm can be used to find the required load to bend a SMA material in order to use the SME to perform the closing motion of a gripper jaw.

4.4. OUTPUT FORCE AND BEAM DISPLACEMENT RELATIONSHIP.

The SME is dependent on the deformation (tip displacement) of the beam. As a consequence, the amount of output force that a beam can deliver is also dependent on the displacement of the beam. The numerical model obtained in previous subsections can be used to estimate the output force that the beam can deliver based on the total tip displacement of the beam. To do this, a numerical simulation is set up including an obstruction object that is initially separated from the beam. The obstruction object has an elastic modulus similar to structural steel ($E = 210$ GPa) so that it does not deform upon contact with the beam. The steps of the numerical simulation performed in ANSYS Workbench are described below:

0. The obstruction is initially separated from the beam, and there is no load on the tip of the beam (Figure 4-24.a). The temperature of the beam is below M_f , ensuring that the SMA material is in the martensite phase.
1. The beam is displaced by a vertical force on the tip of the beam, the obstruction is still far from the tip of the beam (Figure 4-24.b). The temperature of the beam is still below M_f . For this study, the magnitude of the input force and its direction are not relevant.
2. The obstruction moves closer to the beam, at a 2 mm distance from the tip to ensure that the two objects will collide (Figure 4-24.c). The beam does not move since the temperature is still below M_f .
3. The temperature of the beam increases past the austenite phase transformation temperature A_f , however, it does not move due to the presence of the obstruction, instead, it generates a reaction force on the obstruction (Figure 4-24.d).

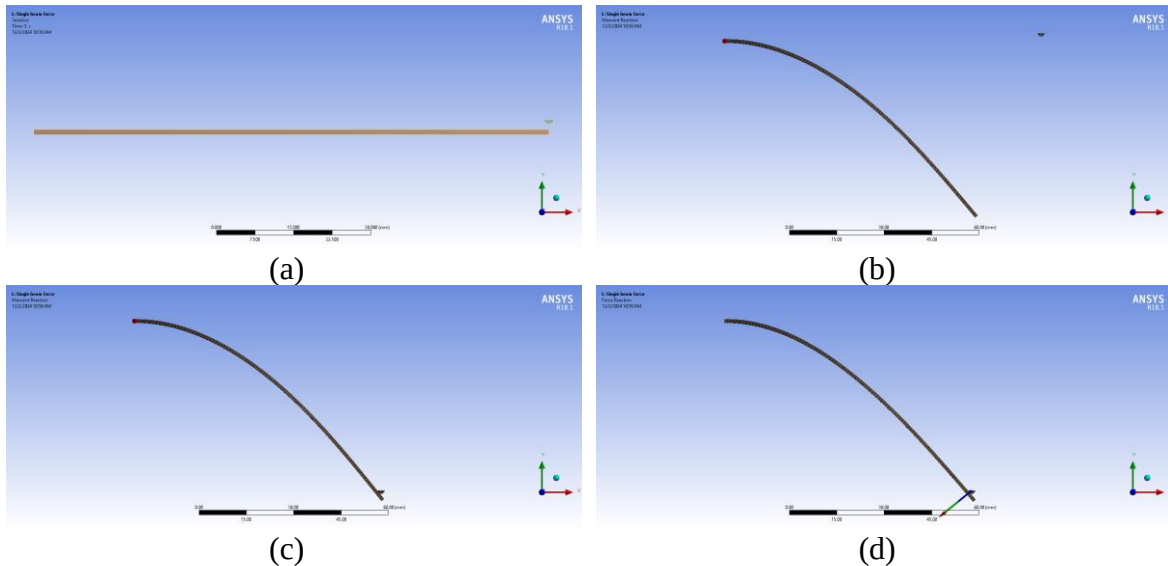


Figure 4-24.- ANSYS Workbench simulation steps to estimate the output force of the beam: (a) initial position, (b) SMA deflection, (c) obstruction position, (d) output force measurement.

This process is repeated for different total beam-tip displacements, and the results are shown in Figure 4-25.

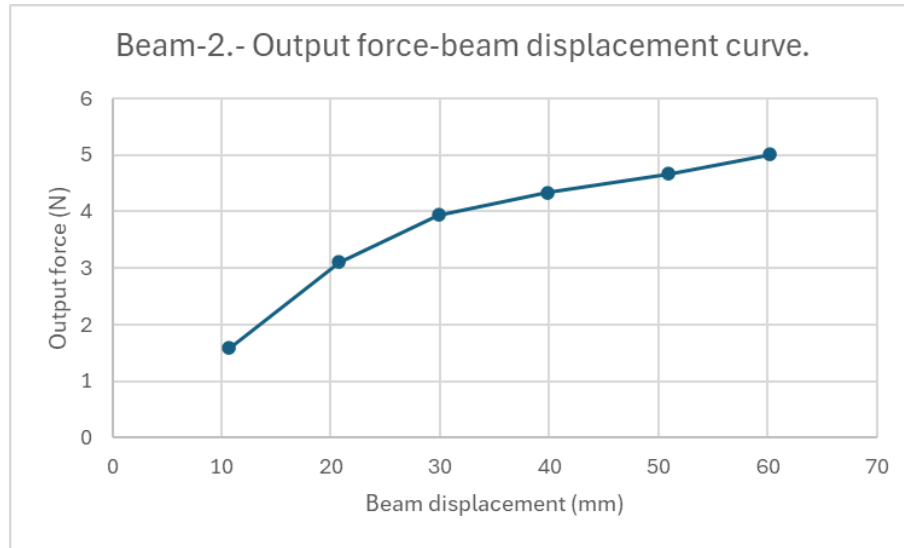


Figure 4-25.- Output force vs total beam tip displacement curve for beam-2, based on the numerical simulation model.

With this analysis, it is possible to estimate the required total beam tip displacement to generate a specific output force. Another important aspect to consider is that the output force is generated in the normal direction of the SMA beam surface, as demonstrated by the reaction force of the obstruction shown in Figure 4-26.

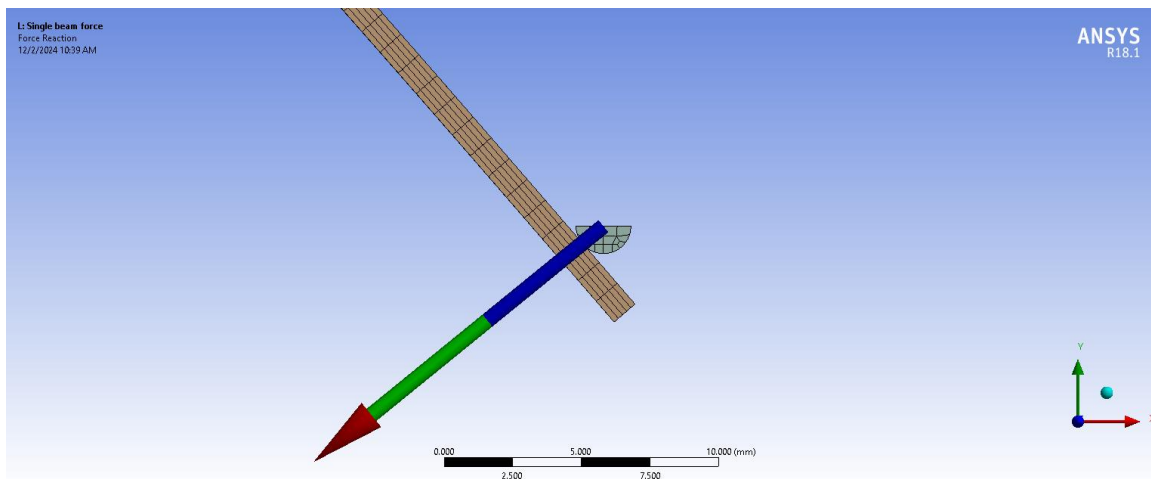


Figure 4-26.- Output force reaction direction.

The direction of the output force should be considered by the gripper design to ensure that the grasping force direction is pointing towards the center of the gripper.

4.5. OUTPUT FORCE RELATION TO THE DIRECTION OF THE INPUT FORCE.

In the previous section the relationship between the total beam-tip displacement and the output force provided by the SME was discussed. The effect of the direction of the input

force is analyzed here to quantify its effect on the output force. To do this, consider the two load cases shown in Figure 4-27.

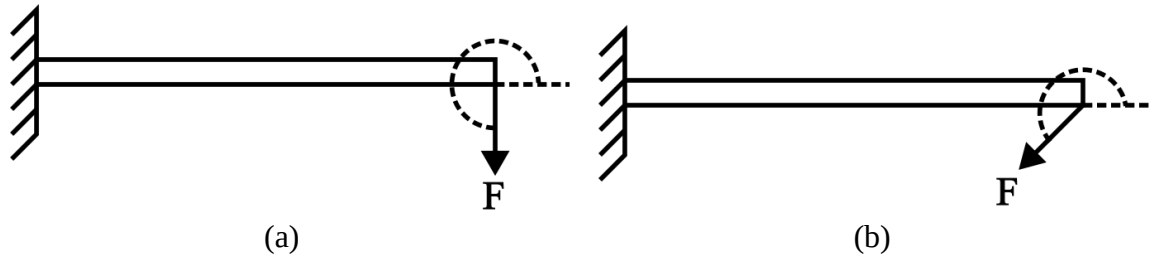


Figure 4-27.- Input force direction at: (a) 270 degrees and (b) 225 degrees.

As shown in Figure 4-27.a, the input force is set to an angle of 270 degrees, and the objective is to calculate the effect, if any, on the output force when modifying the input force angle to 225 degrees, as shown in Figure 4-27.b. This effect is measured in terms of the beam displacement and the output force that the beam can deliver for the same displacement in both cases. The analysis is performed by using the same methodology shown in Figure 4-24.

As shown in Figure 4-28, the input force load angle has a very negligible effect on the output force that the beam can deliver to an object. The measured output force values have an average difference of 0.09 N.

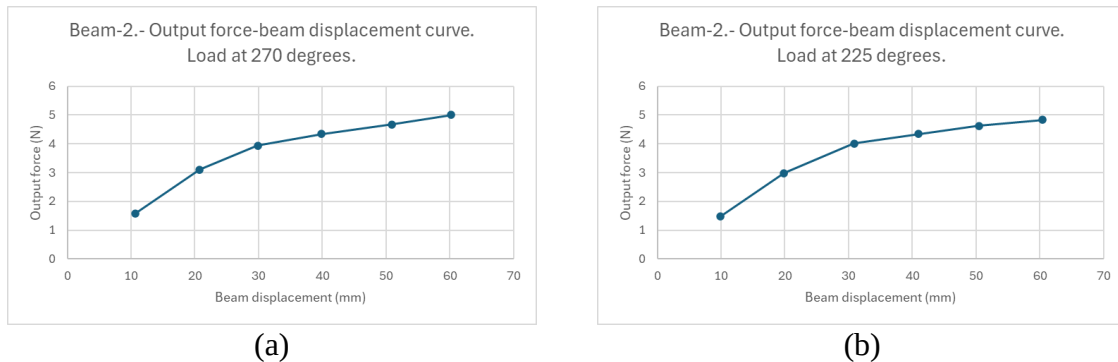


Figure 4-28.- Output force-beam displacement curve for an input load at: (a) 270 degrees and (b) 225 degrees.

Based on the observed behavior, it is possible to calculate a specific required displacement that would allow the SMA actuator to deliver a specific output force, that can be utilized as a grasping force. Furthermore, the required displacement can be obtained regardless of the input force angle since it has a negligible effect on the output force. This information is useful for the formulation of the optimization algorithm in the next section.

5. IMPLEMENTATION OF OPTIMIZATION TECHNIQUES FOR THE DESIGN OF COMPLIANT SHAPE MEMORY ALLOY-BASED GRIPPER.

5.1. THE OPTIMIZATION PROBLEM OBJECTIVES AND REQUIREMENTS.

The SME is triggered to perform the closing motion and provide force to the gripper jaws to grasp objects. It is important to note that the SME does not trigger unless the SMA actuators are deformed so that there is a phase transformation from twinned to detwinned martensite, in this case, the gripper actuators are in the form of cantilever beams.

Another important aspect of using SMAs as actuators is that the output force of the cantilever beam increases non-linearly as the displacement of the beam increases. A grasping force for a specific gripper design requires a specific displacement of the beam. Here, the optimization algorithm is used alongside the PRBM to calculate the required input force F_{in} at a low temperature (Figure 5-1.a) in order to have a specific output force for each beam at high temperatures (Figure 5-1.b).

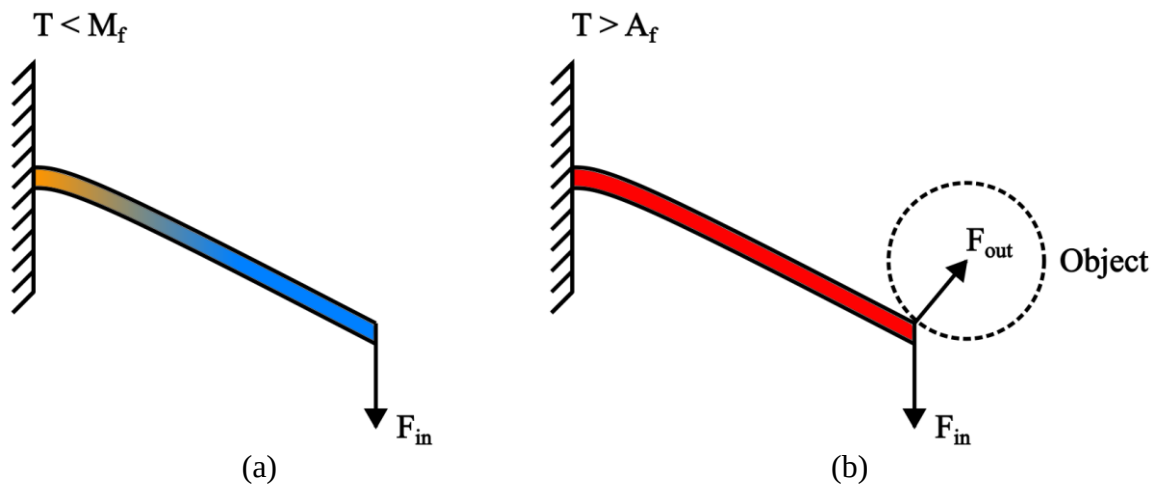


Figure 5-1.- Half of the gripper, showing: (a) Low temperature beam displacement and (b) Force applied to an object at high temperatures.

Until this point, the beam has always been shown initially horizontal. This leads to two possible problems. First, the output force F_{out} is applied to an object at an angle (Figure 5-1.b). Second, with this configuration, the beam output force is dependent on the stroke of the beam. This is because as the beam displaces, the more output force it can provide, this means that to have a high output force, a large stroke is required. The stroke is defined as the available range of motion of each jaw. For two identical gripper jaws, the total stroke would be the sum of the top actuator stroke and the bottom actuator stroke. Furthermore, the stroke may not increase linearly since the SME is a non-linear effect, however, the main focus of this thesis project is to analyze the output force that the gripper can deliver, for this reason, the stroke will not be analyzed. The current design is shown in Figure 5-2.

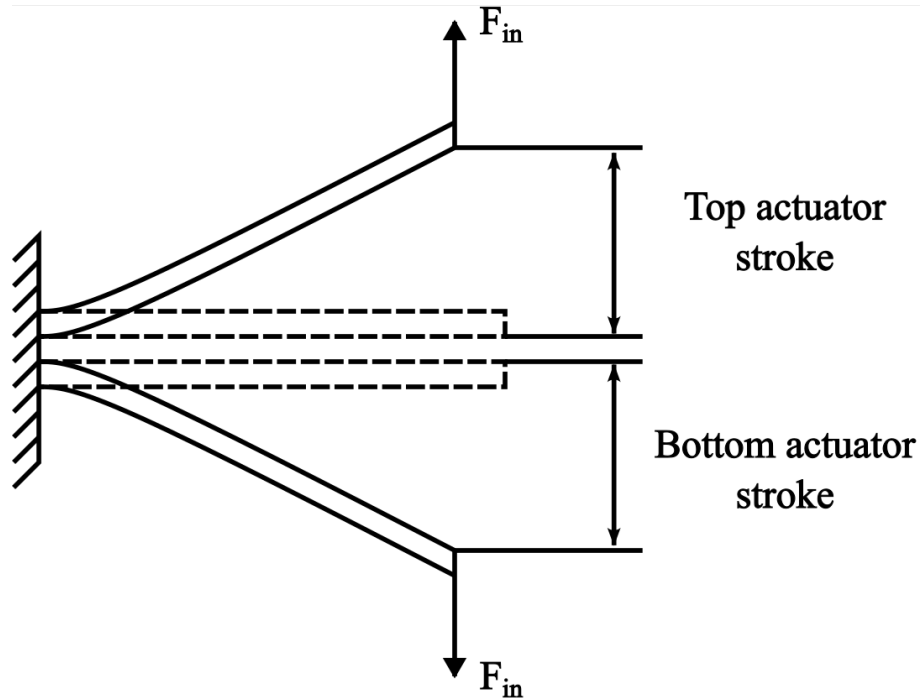


Figure 5-2.- Initial SMA Gripper design.

Having an output force dependent on the stroke is not ideal since it would greatly limit the performance capabilities of the gripper. The output force pointing at an angle would also make it harder for the gripper to grasp objects, since it would be grasping and at the same time pushing the object out. Next, section 5.1.1 shows an alternative gripper design using the same beams in a different arrangement that addresses both of these problems.

5.1.1. ALTERNATIVE GRIPPER DESIGN.

The output force available due to the SME is dependent on the displacement applied to the beam as shown in section 4.4. With the current design, increasing the displacement of the actuators allows the gripper to generate more grasping force. A new design is proposed in this section to have an independent output force from the distance between the fingertips. Rotating the base of each actuator gives room for more displacement of each beam, increasing the output force of the actuators. At the same time, output force becomes independent of distance between the fingertips since each beam is first displaced and then positioned in the open position, as opposed to the previous design where the beams are initially close to each other and the input force opens the gripper jaws.

The proposed gripper design works as shown next, by three main stages:

In the first stage, the SMA actuators are positioned at an angle instead of being horizontal, this will allow for more displacement, resulting in a higher grasping force. At this point there is no input force on the tip of the beams and the temperature of the actuators is below M_f (Figure 5-3).

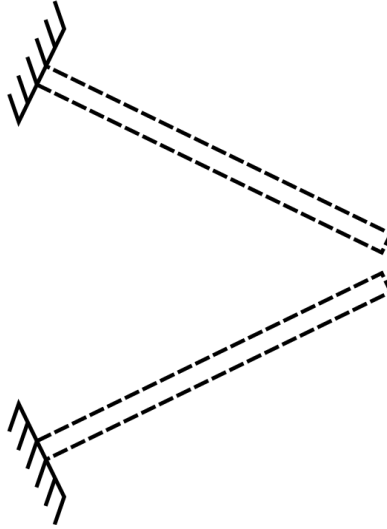


Figure 5-3.- Gripper design initial setup.

In the second stage, the SMA actuators are loaded so that they displace at a temperature below M_f . This allows the phase transformation from twinned martensite to detwinned martensite. After applying the force, the beams can be positioned further away to allow the gripper to pick up larger objects (Figure 5-4.a) or closer to each other for smaller objects (Figure 5-4.b). The input force allows the two actuators to deflect and adopt the open position of the gripper. This is considered the starting position of the gripper, since the beams will always return to this position due to the input force after their temperature is lowered, in this case, the input force is given by a bias spring.

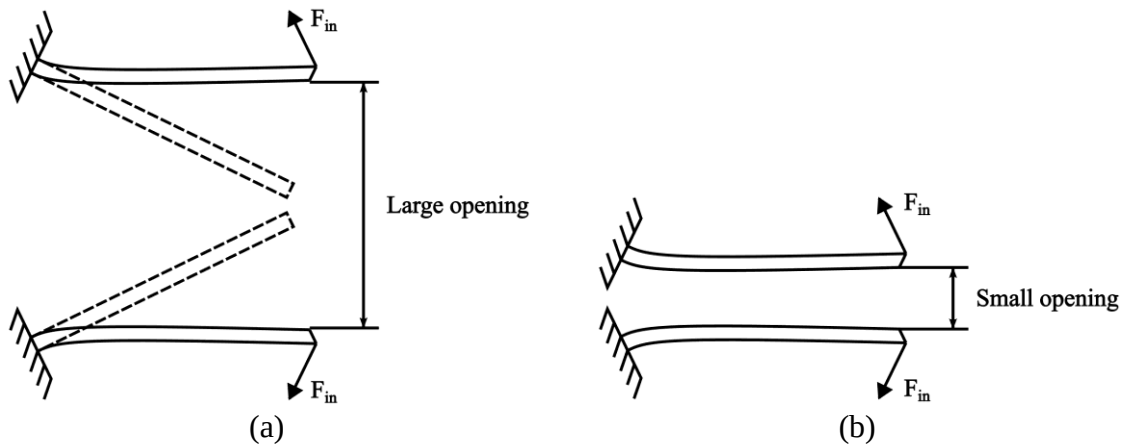


Figure 5-4.- Initial position of the SMA-based gripper, allowing for two different openings: (a) large opening and (b) small opening.

In the third stage, the temperature of the material increases past A_f and the two actuators begin the phase transformation from detwinned martensite to austenite. The beams perform the closing motion since the material is trying to return to its initial position from stage one. However, if there is an object in the recovery trajectory, the actuators start to apply a force onto the object, this is the grasping force F_{out} of the gripper. This stage is

shown in Figure 5-5. With this design, the output force is acting towards the center of the gripper due to the rotation of the actuators.

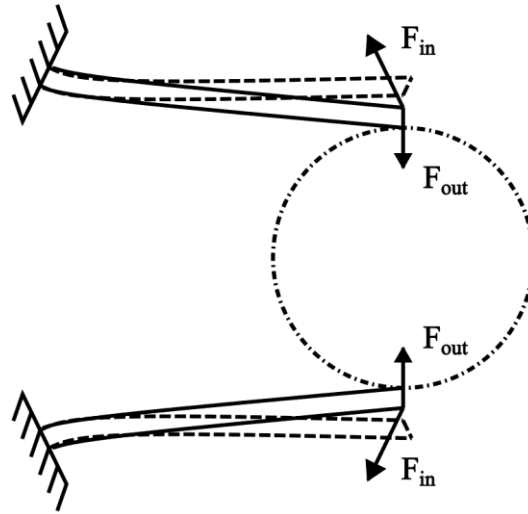


Figure 5-5.- Grasping position of the SMA-based gripper due to the SME.

In the final stage, as the temperature of the SMA actuators drops below M_f , the output force F_{out} decreases to 0 and the SMA actuators return to their initial position in stage two due to the input force F_{in} provided by the bias springs (not shown).

The objective of the optimization algorithm is to find the required input force F_{in} , the input force angle φ and the required beam angle η . The meaning of these variables is shown in Figure 5-6. The optimization constraints are also defined using these variables. For example, the angle of the beam η should range from about 10 to 80° to avoid having a completely horizontal or vertical beam, since a horizontal beam would likely not be able to deliver sufficient grasping force, and a vertical beam would be incompatible with the PRBM since it does not consider buckling.

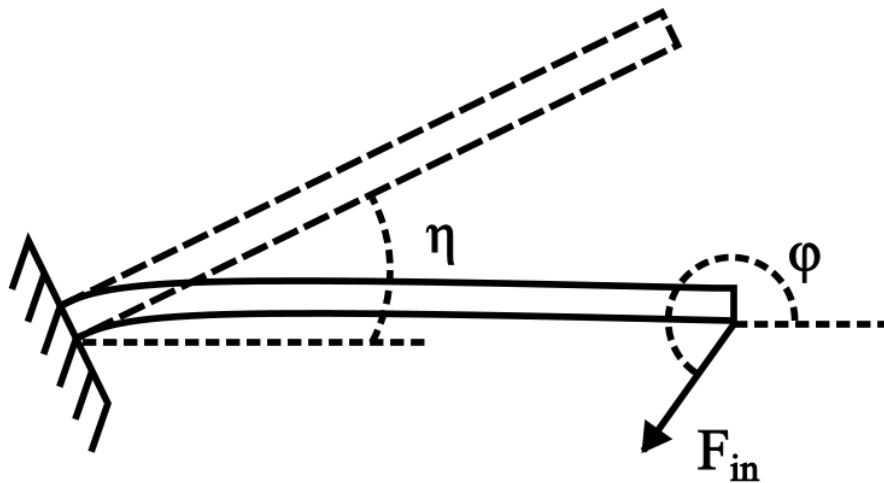


Figure 5-6.- Optimization problem solution variables.

The objective function is to minimize the error for the required total displacement (T_d) to provide a specific output force while also considering that the tip of the beam should be horizontal after applying the force, to ensure that the output force is pointing towards an object positioned in the trajectory of the recovery due to the SME.

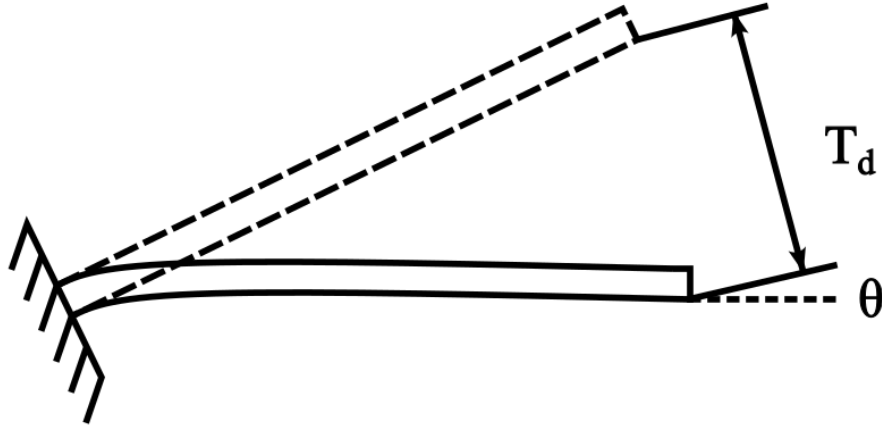


Figure 5-7.- Optimization problem objective function variables.

The PRBM serves as an input to the optimization algorithm, where the objective function and the optimization boundary conditions are defined to find the best possible solution to this problem. The optimization problem is properly defined in detail in section 6.3.5 for a specific case study.

5.2. TWO-BEAM MODEL APPROXIMATION.

In MATLAB, the optimization algorithms *fmincon* and *ga* require a mathematical model to work with. In this regard, the pseudo-rigid body model developed in section 3, can serve as the input for the optimization algorithm. However, there is one problem, and it is that the pseudo-rigid body model does not consider the behavior of an elastoplastic material since it does not involve the elastic limit and the tangent modulus in its calculations.

The proposed solution is to establish a two-beam model to make the pseudo-rigid body model incorporate the parts of the beam that have transitioned into the plastic range. The two beams represent the elastic and the plastic regions of the entire beam.

Consider a SMA cantilever beam with a free end and with no load (Figure 5-8.a), also consider that the temperature of the beam is relatively low, below the austenite start transformation temperature A_s . At this point, the beam is not under any stress, then, the entire beam presents an elastic modulus of the martensite phase E_m .

Afterwards, a force load F_1 is added to the free end of the beam (Figure 5-8.b). In this stage, the load is barely enough to surpass the elastic limit S_y . In a cantilever beam, the stress concentrates on the fixed end of the beam. This stress concentration allows the SMA material to undergo a stress-induced phase transformation from twinned martensite

to detwinned martensite. Furthermore, this part of the beam begins to transition from a martensite elastic modulus E_m to the tangent elastic modulus E_t .

As the load increases from F_1 to F_2 (Figure 5-8.c), the stress continues to concentrate, and the surrounding area starts to transform from E_m to E_t . At this final stage, the beam has two regions. In one region, marked with orange, the stress concentration has transitioned the material from an elastic modulus E_m to the tangent modulus E_t . In the other region, marked with blue, the stress is not greater than the elastic limit S_y , which means that the elastic modulus E_m is still present. The corresponding stress-strain graph represents the two elastic modulus color guides for all stages (Figure 5-8.d).

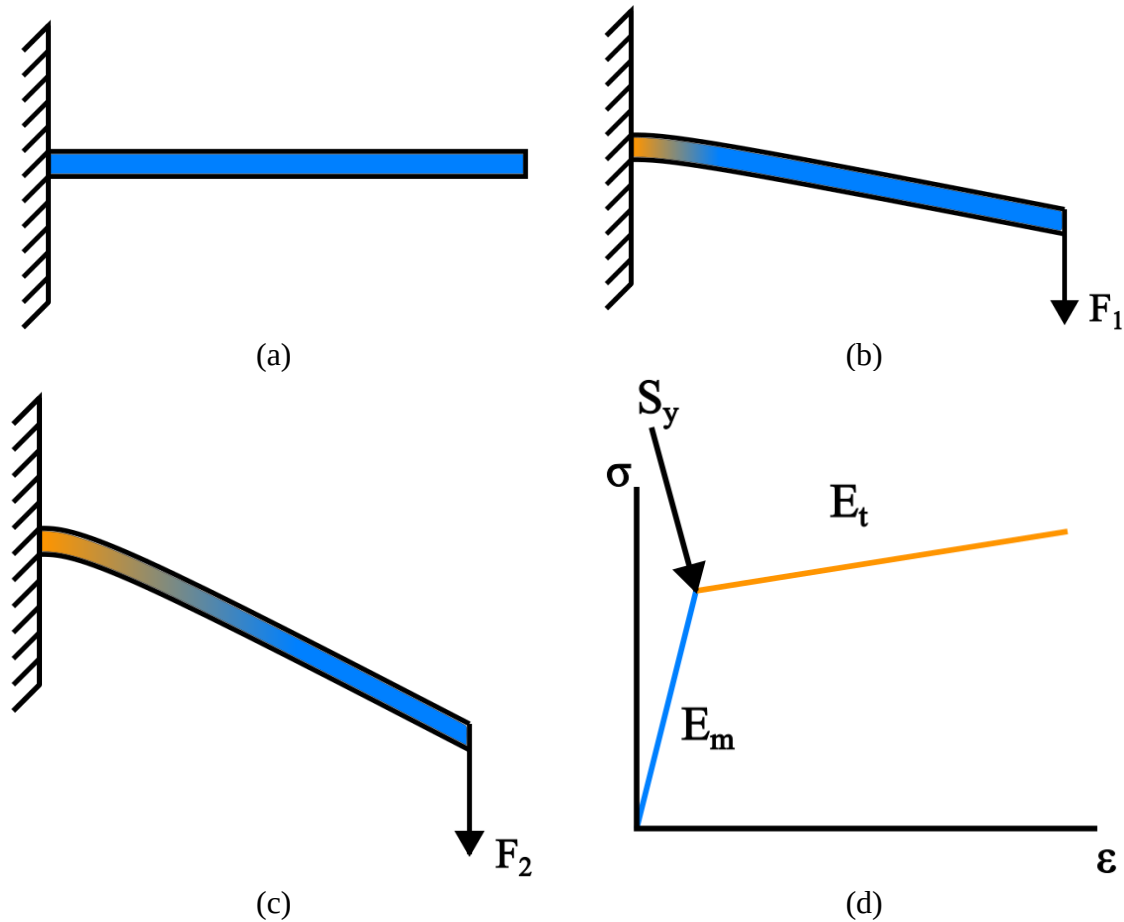


Figure 5-8.- Cantilever beam stress concentration with: (a) no load, (b) F_1 load, (c) F_2 full load, and (d) corresponding stress strain graph.

Based on this behavior, the PRBM can be modified to incorporate the tangent modulus by analyzing the beam as two coupled beams. This is because the PRBM developed in this thesis project has been modified to allow for the analysis of various beams connected in series with varying geometric dimensions and elastic modulus. In this case, the beam connected to the fixed has an elastic modulus equal to the tangent modulus E_t . The other beam is connected to the end of the first beam and the load is added to its free end, as shown in Figure 5-9.

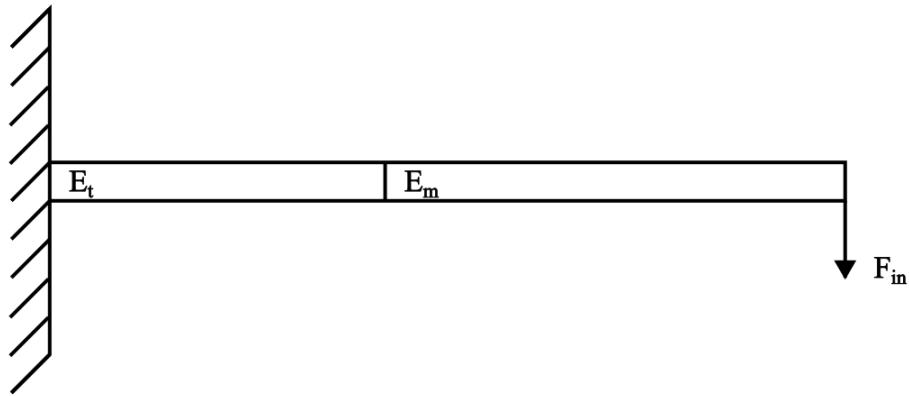


Figure 5-9.- Two-beam model.

This model is usable along with the PRBM MATLAB implementation shown in chapter 3. It does not require the calculation of the stress induced by the load. Instead, it incorporates the tangent modulus and the elastic modulus as two different sections of the entire beam. However, since it does not incorporate beam stress calculations, the length of each beam has to be input manually.

5.2.1. TWO-BEAM MODEL LENGTH RATIO.

The length of each beam is based on how much the beam has transformed from elastic to plastic behavior. This, as stated in section 5.2, depends on the stress concentration of the beam. Since the PRBM does not calculate stresses, a more direct approach is to create a simulation using the SMA material model found in section 4.3. The results are then compared to a different model, where the SMA beam is swapped by the two-beam model. The length of each beam is adjusted so that the total length of both beams is equal to the length of the SMA beam. Afterwards, the force displacement curve of each two-beam model with varying length ratios are compared with the SMA beam model.

For this example, the SMA beam is modeled in ANSYS Workbench, consisting in a horizontal cantilever beam with a 10 N vertical force load, where the beam has a length of 100 mm. Then, the same load is applied to two beams connected in series as shown in Figure 5-9. The length of each beam is varied by a ratio of 0:100, 10:90, 20:80, 70:30 and 40:60, where the 0:100 ratio means that the beam has a E_t section of 0 mm in length and a E_m section of 100 mm in length, similarly, the 10:90 ratio means that the E_t section is 10 mm long and the E_m section is 90 mm long, the same logic applies to the other ratios. The force-displacement curves are shown in Figure 5-10.

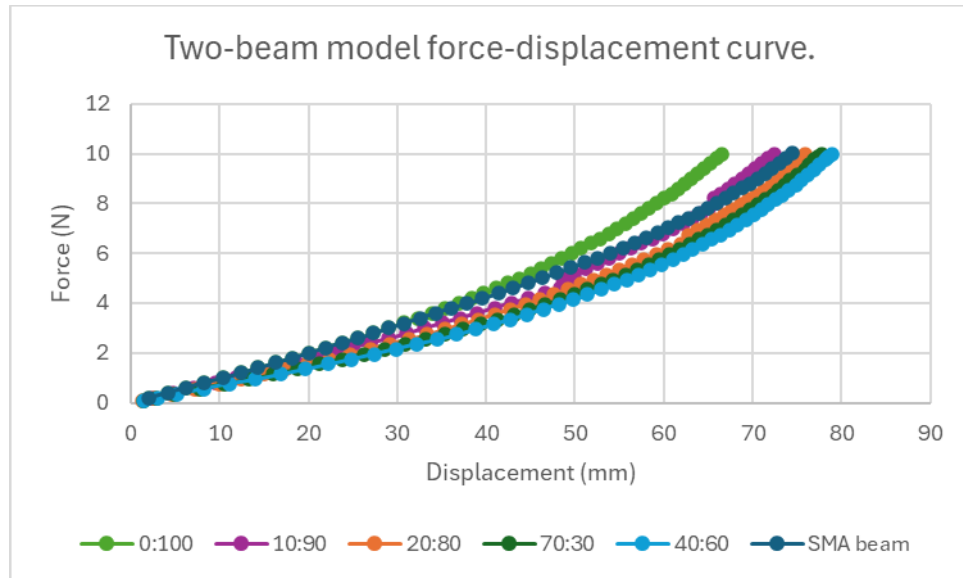


Figure 5-10.- Force-displacement curve of the SMA beam behavior compared to the two-beam model using different ratios.

As shown in Figure 5-10, the two-beam models with ratios 20:80, 70:30 and 40:60 fall below the SMA beam model. These ratios cannot be used with the PRBM and the optimization algorithm because they do not accurately represent the behavior of the SMA beam. Figure 5-11 shows only the remaining models compared to the SMA beam model force-displacement curve.

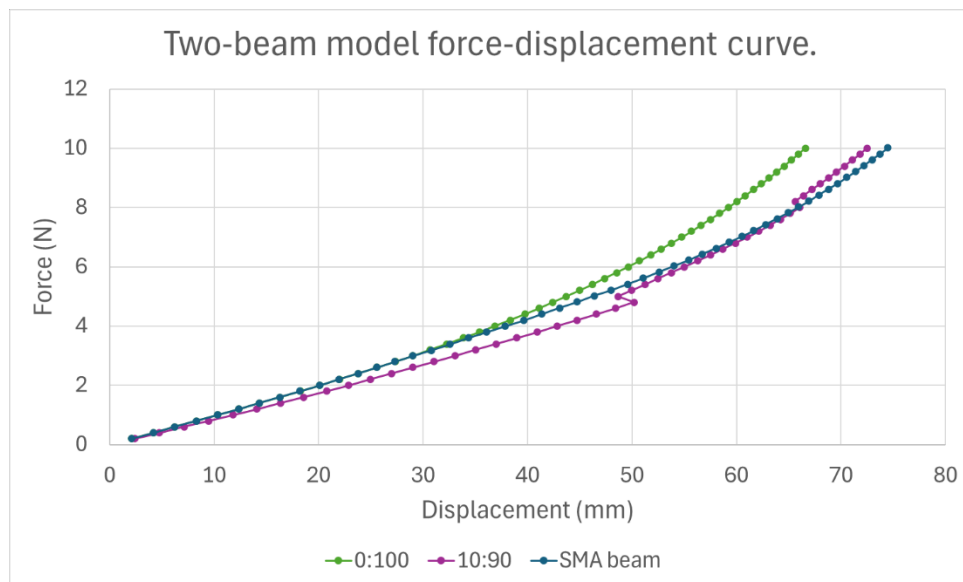


Figure 5-11.- Force-displacement curve of the SMA beam model compared to the two-beam model using ratios: 0:100 and 10:90.

As shown in Figure 5-11, the two-beam model with a ratio of 0:100 has a similar behavior to the SMA beam model for displacements ranging from 0 to approximately 40

mm. Similarly, the two-beam model with a ratio of 10:90 is better when the displacement ranges from approximately 50 mm to 65 mm.

Considering that the total tip displacement T_d is part of the objective function, it means that it is a known variable, for this reason, in this example, if $0 \leq T_d \leq 40$ mm, then the two-beam model with a ratio of 0:100 should be used. Otherwise, if $50 \leq T_d \leq 65$ mm, then the two-beam model with a ratio of 10:90 should be used. Alternatively, if T_d does not fall into any of the two ranges, more numerical simulations should be conducted to find the appropriate two-beam model ratio for that particular case.

It is important to consider that the two-beam model also depends on the angle of the load relative to the beam. For example, consider the cantilever beam rotated by 45 degrees counterclockwise shown in Figure 5-12.

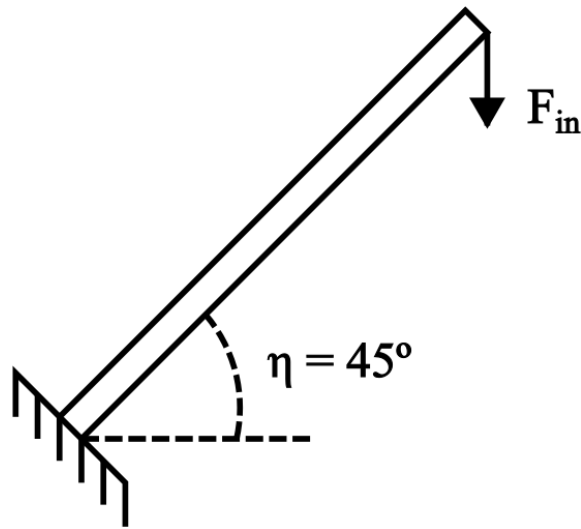


Figure 5-12.- Cantilever beam rotated by 45 degrees.

The force-displacement curve of the SMA model is compared to the curves for the two-beam model with a ratio of 0:100 and 10:90 in Figure 5-13.

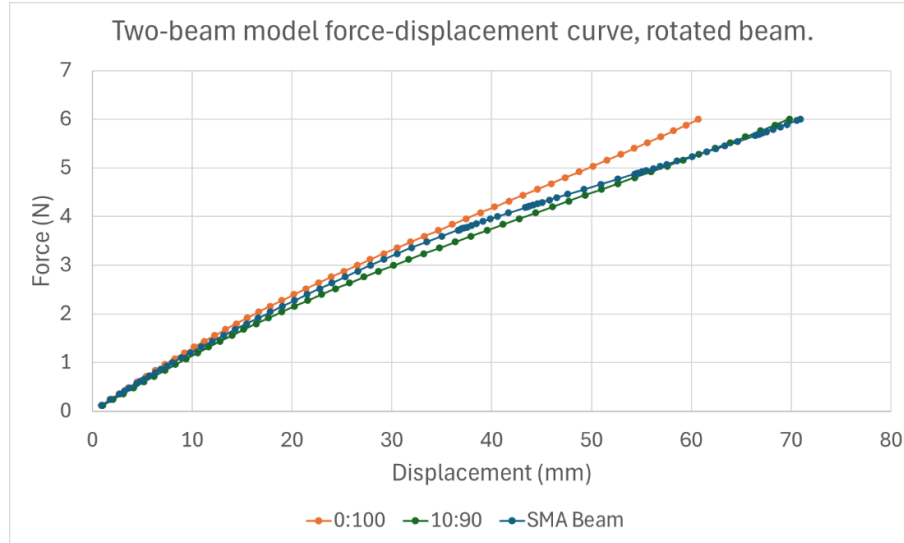


Figure 5-13.- Force-displacement curve of the SMA beam model compared to the two-beam model using ratios 0:100 and 10:90 with a beam rotation of 45 degrees.

As shown in Figure 5-13, the rotation of the beam has an effect on the accuracy of the model, specifically, the resulting curve for the two-beam model with a ratio of 10:90 becomes much more similar to the SMA beam model for the entire range, however, it is less accurate for a displacement range of 0 to 50 mm as shown in Figure 5-11. In this case, the two-beam model with a ratio of 0:100 is not as affected by the rotation of the beam, since it is still the better option for a displacement in the range of 0 to 40 mm.

The effect of the input force F_{in} is dependent on the input force angle φ , relative to the beam rotation angle η as shown in Figure 5-11 and Figure 5-13. Only these two cases were analyzed considering that the input force cannot be at an angle φ higher than 270 degrees since it would locate the spring bases in front of the gripper actuator tips, which could be problematic. A more in-depth analysis is performed in sections 6.3.4 and 6.4.1 based on two different case studies, where the effect of the input load angle is analyzed in more detail.

The two-beam model approximation allows the use of the PRBM, since the model does not require the calculation of the stress in the beam, instead, the required displacement is used to select the appropriate two-beam model approximation. After selecting the appropriate two-beam model, the optimization algorithm can solve the optimization problem.

5.3. OPTIMIZATION PROBLEM FORMULATION.

In section 2.9, two optimization algorithms were introduced: *fmincon* and *ga*. Both of these algorithms have their own advantages and disadvantages. As a first step, it is important to analyze the requirements of each optimization algorithm.

As a reminder, *fmincon* is defined in MATLAB as shown below:

$$x = fmincon(fun, x_0, A_{ineq}, b_{ineq}, A_{eq}, b_{eq}, lb, ub, nonlcon, options)$$

Where:

fun: objective function of the optimization problem.

x_0 : starting point for optimization variables.

A_{ineq} : inequality constraints coefficients matrix.

b_{ineq} : inequality constraints results vector.

A_{eq} : equality constraints coefficients matrix.

b_{eq} : equality constraints results vector.

lb : lower bound for each problem variable.

ub : upper bound for each problem variable.

nonlcon: non-linear constraints of the problem.

The objective function formulation, as mentioned in section 5.1.1, is based on the required total displacement of the beam. The objective function also needs to consider the inclination of the tip of the beam, specifically, the inclination must be as close to 0 as possible, to ensure that the output force delivered by the gripper does not point outwards, pushing the object (refer to Figure 5-5). Two objective functions can be created based on these requirements, where the errors are to be minimized and are represented by equations (5-1) and (5-2).

$$\left| \frac{T_{di} - T_d}{T_d} \right| * 100 \quad (5-1)$$

$$\left| \frac{\theta_i - \theta_{tip}}{\theta_{tip}} \right| * 100 \quad (5-2)$$

Where T_d refers to the required total displacement, T_{di} is the total displacement of the current iteration of the optimization algorithm, θ_{tip} is the required beam tip angle and θ_i refers to the tip angle of the current iteration of the optimization algorithm. Equations (5-1) and (5-2) are based on the absolute percentage error formula (2-15).

The optimization algorithm *fmincon* only requires a single objective function. The advantage of using the absolute percentage error is that it cannot give negative values, because of this, equations (5-1) and (5-2) can be summed to create a single objective function. Ideally, if both the tip displacement and the tip angle are equal to their required values, the objective function value becomes zero, otherwise, the optimization algorithm will try to minimize the objective function. The problem with this approach is that the optimization algorithm cannot prioritize any of the two objectives. This may lead to undesirable solutions. The resulting objective function to be minimized is represented by equation (5-3).

$$fun = \left| \frac{T_{di} - T_d}{T_d} \right| * 100 + \left| \frac{\theta_i - \theta_{tip}}{\theta_{tip}} \right| * 100 \quad (5-3)$$

One of the differences between *fmincon* and *ga* is that *fmincon* requires a starting point x_0 . This starting point is the initial values of all the optimization variables. In section 5.1, the optimization variables were briefly mentioned. To attain the required beam tip displacement, two variables are considered: the input force F_{in} and the input force angle φ . These two variables are responsible for the displacement of the beam, calculated by the PRBM using equations (3-7) to (3-13). It is important to note that for this thesis project, even if a moment load can be used for the calculations, it is simpler to not include it for experimental validation of the optimization algorithm in section 6.3.7. To ensure that the beam tip inclination is as close to 0 as possible, the optimization algorithm also needs to calculate the beam initial rotation η . Refer to Figure 5-6 for a visual representation of the optimization variables F_{in} , φ and η . Based on the previous information, the optimization variables initial values x_0 are represented in MATLAB by a vector, as shown in equation (5-4).

$$x_0 = [F_0 \quad \varphi_0 \quad \eta_0] \quad (5-4)$$

The starting point x_0 , is useful if there is a rough idea of the required load to minimize equation (5-3).

The matrix A_{ineq} and the vector b_{ineq} are used in *fmincon* to consider equality constraints. These constraints are defined as shown in equation (5-5).

$$\begin{aligned} A_{ineq11}x(1) + A_{ineq12}x(2) \dots A_{ineq1m}x(m) &= b_{ineq1} \\ &\vdots \\ A_{ineqn1}x(1) + A_{ineqn2}x(2) \dots A_{ineqnm}x(m) &= b_{ineqm} \end{aligned} \quad (5-5)$$

After defining the equality constraints, the matrix A_{ineq} and the vector b_{ineq} can be introduced in MATLAB as shown in equation (5-6).

$$A_{ineq} = \begin{bmatrix} A_{ineq11} & \dots & A_{ineq1m} \\ \vdots & \ddots & \vdots \\ A_{ineqn1} & \dots & A_{ineqnm} \end{bmatrix}, b_{ineq} = [b_{ineq1} \quad \dots \quad b_{ineqm}] \quad (5-6)$$

Similarly, the inequality constraints are defined as shown in equation (5-7).

$$\begin{aligned} A_{eq11}x(1) + A_{eq12}x(2) \dots A_{eq1m}x(m) &= b_{eq1} \\ &\vdots \\ A_{eqn1}x(1) + A_{eqn2}x(2) \dots A_{eqnm}x(m) &= b_{eqm} \end{aligned} \quad (5-7)$$

Which would result in matrix A_{eq} and vector b_{eq} as shown in equation (5-8).

$$A_{eq} = \begin{bmatrix} A_{eq11} & \dots & A_{eq1m} \\ \vdots & \ddots & \vdots \\ A_{eqn1} & \dots & A_{eqnm} \end{bmatrix}, b_{eq} = [b_{eq1} \quad \dots \quad b_{eqm}] \quad (5-8)$$

In this thesis project $x(1)$, $x(2)$ and $x(3)$ would be related to the optimization variables F_{in} , φ and η , respectively, however, there are no equality constraints nor inequality constraints. As a consequence, the matrices A_{ineq} and A_{eq} and the vectors b_{ineq} and b_{eq} are not used and can be defined as empty brackets in the MATLAB code.

The optimization variables used in equation (5-4) sometimes will be bounded. In this thesis project the optimization variables are bounded depending on the problem requirements.

For example, refer to Figure 5-13 and suppose that the required total displacement T_d is 60 mm. For the input load F_{in} , a good approach is to bound the input force based on the FEM analysis, in this example, it is known that the total displacement can be reached with an input force of about 5.2 N. Furthermore, the analysis shown in Figure 5-13 was performed with a beam rotation η of 45 degrees. Based on the previous information, and using a factor of 1.5 for the upper bound and 0.5 for the lower bound, the bounded variables are expressed as shown in equations (5-9) and (5-10).

$$2.6 \leq F_{in} \leq 7.8 \quad (5-9)$$

$$22.5 \leq \eta \leq 67.5 \quad (5-10)$$

A final thing to consider is that the analysis shown in Figure 5-13 is performed using a vertical load, pointing downwards, in this case, it would be best to consider a force angle φ that would not interrupt in front of the gripper, this results in an upper limit for the force angle of 270 degrees, for the lower limit, it is more complicated to define since it would depend on the rotation angle of the beam. Consider for example the lower limit of the rotation of the beam. The force angle should be bound so that the force cannot go through the beam, as shown in Figure 5-14.

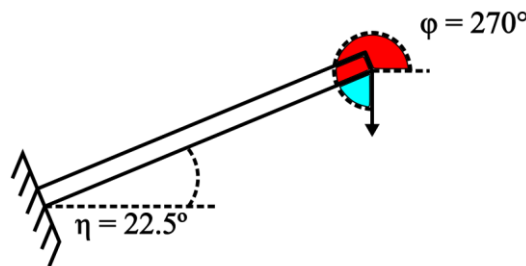


Figure 5-14.- Available force load angle (marked with blue) when using the lower bound rotation of the beam.

In this case, the lower bound for the force load should be defined as 202.5 degrees, since the allowable force load angle range, marked with blue, ranges from 202.5 degrees to 270 degrees. On the other hand, when using the upper bound for the rotation of the beam, the allowable force load angle range is severely reduced.

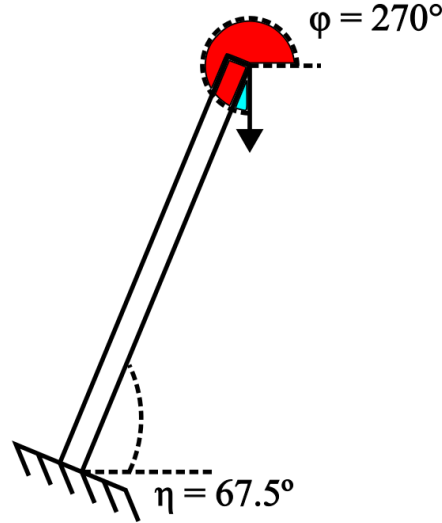


Figure 5-15.- Available force load angle (marked with blue) when using the upper bound rotation of the beam.

As shown in Figure 5-15, the input force load angle available range, marked with blue, has been reduced. The resulting range is bound from 247.5 degrees to 270 degrees. It is not beneficial to have very tight ranges for any variable as it could limit the optimization algorithm available options and affect the results. For this reason, the broader range for the force load angle φ is used for the lower and upper limits.

$$202.5 \leq \varphi \leq 270 \quad (5-11)$$

After bounding the optimization variables, they can be introduced as vectors into MATLAB as shown in equation (5-12) and (5-13).

$$lb = [F_{min} \quad \varphi_{min} \quad \eta_{min}] \quad (5-12)$$

$$ub = [F_{max} \quad \varphi_{max} \quad \eta_{max}] \quad (5-13)$$

For the example given above, the vectors lb and ub are defined as shown in equations (5-14) and (5-15).

$$lb = [2.6 \quad 22.5 \quad 202.5] \quad (5-14)$$

$$ub = [7.8 \quad 67.5 \quad 270] \quad (5-15)$$

It is important to note that if the optimization algorithm is not capable of finding an optimal solution, increasing the range for the optimization variables may give better results.

The argument *nonlcon* refers to nonlinear constraints, in MATLAB, it is defined differently to equality or inequality constraints. This argument will not be used in this thesis project since there are no nonlinear constraints related to the optimization variables.

The argument *options* is used to modify different parameters of the optimization algorithm. This argument will not be used either, since the optimization algorithm works as expected.

On the other hand, the optimization algorithm *ga* is defined in MATLAB as shown below.

$$x = ga(fun, n_{vars}, A_{ineq}, b_{ineq}, A_{eq}, b_{eq}, lb, ub, nonlcon, intcon, options)$$

Where:

fun: objective function of the optimization problem.

x_0 : starting point for optimization variables.

A_{ineq} : inequality constraints coefficients matrix.

b_{ineq} : inequality constraints results vector.

A_{eq} : equality constraints coefficients matrix.

b_{eq} : equality constraints results vector.

lb: lower bound for each problem variable.

ub: upper bound for each problem variable.

n_{vars} : number of optimization variables.

nonlcon: non-linear problem constraints.

As shown, it shares many arguments with the optimization algorithm *fmincon*. One key difference is that it does not require a starting point x_0 , this is because the genetic algorithm starts by creating a random initial population, then the algorithm creates a sequence of new populations based on their fitness value. The randomness of the algorithm eliminates the necessity of a starting point.

Another difference is that this algorithm requires an argument called n_{vars} , this argument simply refers to the number of optimization variables for the problem, in this case, $n_{vars} = 3$, since there are only three optimization variables that the algorithm needs to evaluate to minimize the objective function *fun*.

5.4. OPTIMIZATION ALGORITHM IMPLEMENTATION IN MATLAB.

The MATLAB implementation of the optimization algorithm makes use of the PRBM program. Refer to chapter 3 for an explanation of how the PRBM works and its implementation in MATLAB.

A flow diagram of the optimization program is shown in Figure 5-16 to give a better idea of how the MATLAB code works.

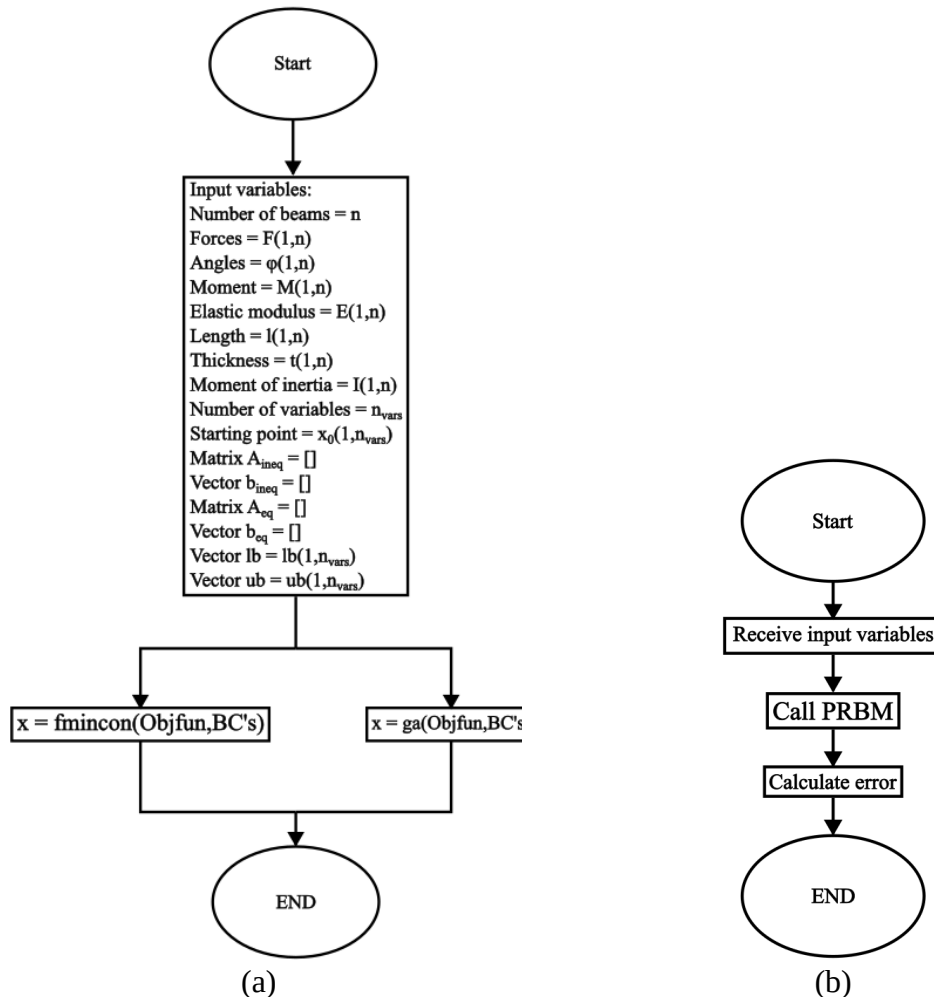


Figure 5-16.- (a) Flow diagram for optimization program implemented in MATLAB and (b) Flow diagram of Objfun function called by optimization program.

The program requires some of the same input variables as the main program from Figure 3-8. In addition, the program also requires the definition of the constraints based on the boundary conditions (if any) and the proper bounds for each optimization variable, as defined in equations (5-4) to (5-15).

The program then splits into two branches, depending on the optimization algorithm to be used. As explained before, *fmincon* is better suited for problems where there is a rough idea of the solution to the problem, since it requires a starting point to solve the problem, while *ga* is more robust and does not require a starting point, however, it could take longer to solve the problem. The variable bounds are then introduced as shown in section 5.3.

Since the two optimization algorithms behave similarly, the function *Objfun* can be used regardless of the optimization algorithm selection. This function receives the input variables and utilizes a similar version of the PRBM program shown in Figure 3-8. The implementation here works the same way as before, however, it is modified so that it

receives the values for F_{in} , φ and η from the optimization algorithm *fmincon* or *ga*, and, since it is now a function, it returns the error calculated using equation (5-3).

The optimization algorithm performs several iterations until the optimization function has been minimized or the number of iterations has reached a certain limit. After that, the resulting plot is displayed to the user and the solution vector appears in the MATLAB Workspace.

6. PROPOSED METHODOLOGY FOR THE DESIGN OF A COMPLIANT SHAPE MEMORY ALLOY-BASED GRIPPER.

6.1. METHODOLOGY FLOW DIAGRAM.

The proposed methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper is shown in Figure 6-1.

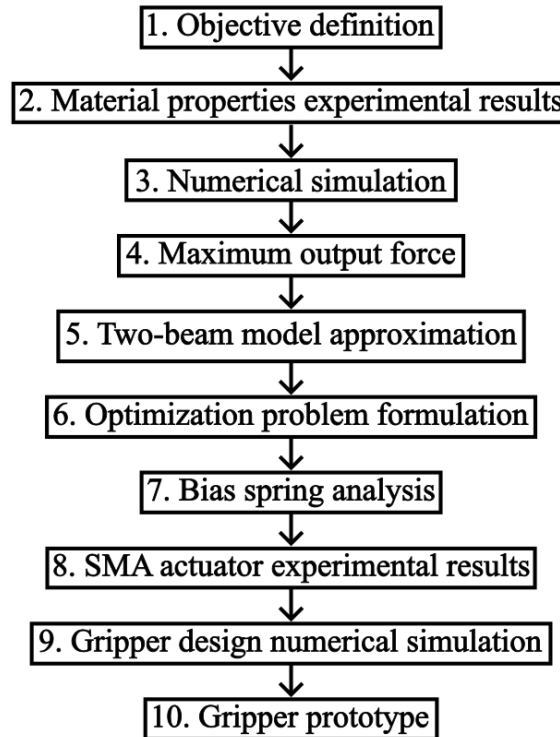


Figure 6-1.- Proposed methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper.

The first step in the proposed methodology is to define the objective output force; this objective should be based on a specific task that the gripper should be able to perform. As previously stated, the objective output force affects the stroke, but with the gripper design proposed in section 5.1.1, the distance between the tip of the gripper jaws can be modified depending on the size of the object to be grasped (Figure 5-4), with the disadvantage that the gripper may not be able to fully close its jaws.

The second step refers to the experimental analysis performed with the objective of calculating the SMA material properties, ideally, a tension test must be performed to determine the elastic modulus for the martensite phase E_m , austenite phase E_a , the tangent modulus E_t and the elastic limit S_y . However, if this is not possible, the material properties can be estimated using the methods explained in section 4.3.

The third step involves creating the numerical simulations required to model the SMA material, where the material properties from the second step are used to represent the experimental behavior of the SMA.

The fourth step is related to calculating the maximum output force of the actuators, to ensure that the objective output force is attainable, if not, one could utilize the numerical simulation model of the actuators in order to find better geometric properties that allow the beam to deliver the required output force. In this step it is important to consider the maximum strain that a SMA can exhibit without creating an unrecoverable deformation, for Ni-Ti alloys, this value can be up to 8% [25].

The fifth step involves the two-beam model approximation, discussed in section 5.2, which serves as a tool for the approximation of the displacement of a SMA beam for step five.

The sixth step uses the two-beam model and optimization algorithms discussed in section 5 to minimize the error of the required total displacement of a SMA beam in order to deliver the objective output force, and to ensure that the tip of the beam ends in an horizontal position to properly deliver the grasping force onto an object.

In the seventh step, the use of a bias spring is implemented in FEM to deliver the beam displacement instead of using an input force, to better represent the real-life functionality of the gripper design.

In the eighth step, the numerical results are reproduced in an experimental test to demonstrate that the optimization algorithm results are correct and that the actuators are capable of delivering the objective output force.

In the ninth step, a numerical simulation is set up demonstrating the functionality of the gripper design.

In the final step, the gripper prototype is built to demonstrate its functionality and the results of the proposed gripper design methodology.

A practical example of the methodology in use is shown in section 6.3, where each step will be performed and further explained as required.

6.2. SHAPE MEMORY ALLOY-BASED GRIPPER PERFORMANCE OBJECTIVE.

In this chapter, a SMA-based gripper will be designed using the tools introduced in chapters 2 to 5. To begin with, a performance objective should be established. In this project, the performance objective is based on a bibliographic analysis of gripper devices, see Table 6-1. An initial analysis was performed, considering various gripper devices, with varying distinctive capabilities. Ganapathy et al. [13] designed a SMA-based gripper, where the grasping force is delivered by SMA wires coupled to a compliant structure. Rojas and Dollar [45] proposed a two-finger topology which takes advantage of an elastic pivot joint between the two fingers that increases the gripper in-hand manipulation capabilities. Xu [46] proposes a gripper design dedicated to micro assembly tasks, with the ability to detect grasping forces and environmental interaction forces. Li et al. [47] designed an underactuated robot hand with 10 degrees of freedom that only

requires 4 input motors, which allow the gripper to grasp different sized objects. Ko [48] proposed an underactuated tendon-driven gripper with exceptional pinching capabilities thanks to a crawler added to one of the two fingers of the design. Chen and Lan [49] designed a force regulation mechanism for pneumatic grippers, allowing them to deliver an adjustable output force to carefully manipulate fragile objects. Backus and Dollar [50] present a three-fingered robotic hand which is actuated by a single motor, thanks to a design that combines prismatic and revolute joint underactuated fingers with passive revolute joints. Similarly, Telegenov et al. [51] presents a three-fingered robotic gripper design actuated by a single actuator, which utilizes a series of gears to actuate the three fingers at the same time. The final design included in Table 6-1 was developed by Kim et al. [52] which consists of a three-fingered gripper with five degrees of freedom with the ability to transition from precise parallel pinching to a compliant grasping mode. Some characteristics relevant to this work of the previously mentioned gripper designs are shown in Table 6-1.

Author	Year	Stroke (mm)	Force (N)	Compliant	Number of outputs	Number of inputs	Dexterity
Ganaphaty Then Mozhi [13]	2023	10	0.38	x	2		
Nicolas Rojas [45]	2016	103	8.9		2	2	x
Qingsong Xu [46]	2017	10.2	0.815	x	2	1	
Xia-Li Li [47]	2018	136	4.89		3	4	x
Tianyi Ko [48]	2020	200	20		2	1	x
Chih-Chieh Chen [49]	2018	20	10	x	2	1	
Spencer Backus [50]	2016	130	60		3	1	
Kuat Telegenov [51]	2014		15		3	1	
Yong-Jae Kim [52]	2020	100	60		3	5	

Table 6-1.- Bibliography analysis of gripper designs.

The column “Stroke” in Table 6-1 refers to the maximum distance between the fingertips. The column “Force” refers to the maximum grasping force that the gripper design is able to deliver to an object. The column “Compliant” specifies if the gripper design obtains its movement entirely or partially due to the effect of a compliant component. The column “Number of outputs” specifies the number of fingers that the gripper design has, and it should be analyzed using the column “Number of inputs”, as the relation between the two determines if the gripper design is underactuated $n_{inputs} < n_{outputs}$, overactuated $n_{inputs} > n_{outputs}$, or neither. At last, the column “Dexterity” specifies if the gripper design is able to perform more movements apart from a closing and opening motion.

The focus of this project is to create a methodology for the design of a SMA-based gripper. The design, introduced in section 5.1.1, should be able to deliver the stroke and force performance characteristics specified by the designer, by following the methodology shown in the following sections. The gripper design is inherently compliant due to the required displacement of the SMA in order to trigger the SME. The number of inputs for the gripper design developed in this thesis project is 2, as it requires two power

supplies to trigger the SME with the help of two Ni-Cr sheets, one for each gripper jaw. Alternatively, if a single power supply can deliver the required current for both Ni-Cr sheets, then the number of inputs can be reduced to 1. Similarly, the number of outputs is 2, as the current gripper design only considers two gripper jaws. In regard to the dexterity of the gripper design, it cannot perform motions differently to the opening and closing of the gripper jaws, so it is not considered dexterous by the definition set in this thesis project.

A comparison of the force-stroke relationship of the various gripper designs from Table 6-1 is shown in Figure 6-2.

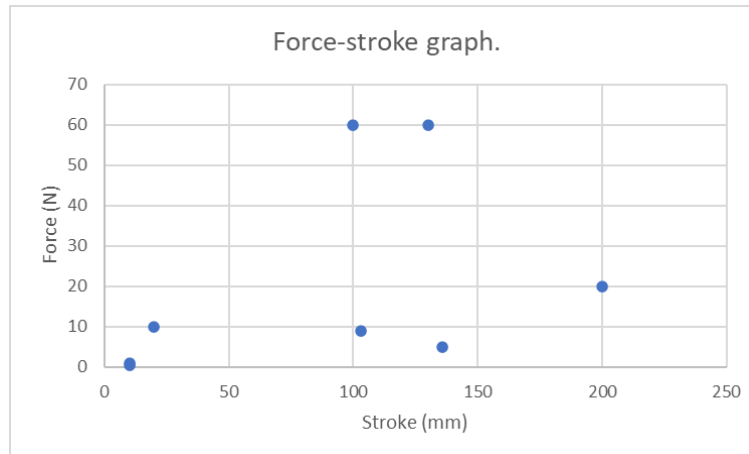


Figure 6-2.- Force-stroke relationship chart for initial research.

As shown in Figure 6-2, the force-stroke relationship from the research shown in Table 6-1 does not follow a specific trend. Considering that the force can reach values higher than 10 N, some more research is performed, this time, the research is focused on gripper designs where the grasping force is not superior to 10 N, since the SMA actuators beam-1 and beam-2 are not able to deliver as much force in normal working conditions. For this reason, the second research is also focused on finding more examples of SMA-based grippers, to have a better comparison in terms of performance, specifically, the stroke and force of the gripper designs. Rodrigue et al. [7] presents a SMA wire-based soft gripper that utilizes a composite polydimethylsiloxane (PDMS) matrix that is flexible enough to allow the SMA wires to perform large deformation while also providing enough rigidity to bring the actuator back to its initial position. Lu et al. [6] designed a SMA wire-based gripper where eight SMA wires are connected in series to achieve a large stroke by incorporating sliding shafts and a bias spring into its design. Wang and Ahn [10] present a SMA-based soft gripper actuator that also incorporates a PDMS matrix and a shape memory polymer (SMP) to add two hinges to each of the three fingers from their design, where the change in temperature of the SMA and SMP is provided by a current passing through Ni-Cr wires. Wang et al. [11] designed a soft planar gripper that also utilizes a SMA wire and a PDMS matrix to perform a hinge-like movement to grasp deformable objects. Liu et al. [53] designed a variable stiffness design of a SMA-based soft gripper composed of three robotic fingers that is capable of grasping compliantly at low stiffness and holding robustly at high stiffness, utilizing paraffin as a variable stiffness material in

order to allow an SMA wire to drive the robotic finger at high temperatures, where the paraffin has a low stiffness and provides the joint with sufficient stiffness at low temperatures to sustain external loads. Huang et al. [54] designed an underactuated compliant gripper, this gripper is not SMA-based, however it falls into the established force range of this research, the gripper has two working modes, namely, two-point grasping and enveloped grasping, and utilizes a PRBM to perform the static analysis of both grasping modes. Liu et al. [55] presents a monolithic two-finger compliant gripper design where optimization methods were utilized to maximize the mechanical advantage of the synthesized mechanism. Petkovic et al. [56] designed an adaptive underactuated compliant gripper with distributed compliance, capable of grasping irregular objects due to its high flexibility. The performance characteristics of the previously presented gripper designs are shown in Table 6-2.

Author	Year	Stroke (mm)	Force (N)
Hugo Rodrigue [7]	2017	60	1.5
Yifan Lu [6]	2019	100	7.5
Wei Wang [10]	2017	80	5.8
Wei Wang [11]	2021	40	0.57
Mingfang Liu [53]	2020	90	5.66
Mingxian Huang [54]	2019	80	6.04
Ganapathy Then Mozhi [13]	2023	10	0.38
Chih-Hsing Liu [55]	2019	35	5
Dalibor Petkovic [56]	2013	60	0.075
Average:		61.67	3.61

Table 6-2.- Bibliography analysis of gripper designs with a maximum grasping force of 10 N.

As shown in Table 6-2, the various gripper designs can deliver an average stroke of 61.67 mm and an average grasping force of 3.61 N. The maximum grasping force shown in Table 6-2 is 7.5 N, while the maximum stroke is 100 mm.

The stroke of the gripper design proposed in section 5.1.1 is calculated based on the initial distance of the tip of each beam, as well as the maximum recovery of each beam due to the SME.

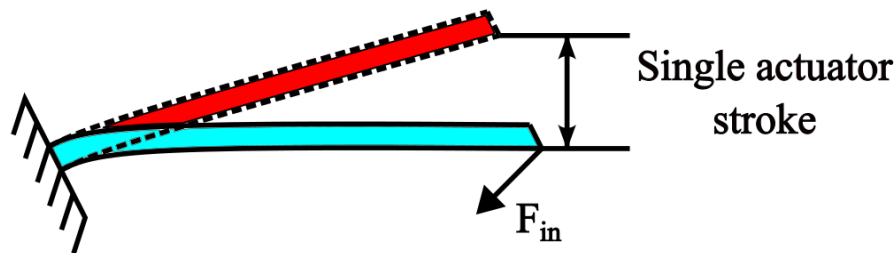


Figure 6-3.- Stroke contribution of a single SMA actuator for the proposed gripper design.

As shown in Figure 6-3, where only the lower actuator is present, the stroke for a single actuator is given by the travelled distance of each beam from the initial position in the martensite phase (colored in blue), to the grasping position in the austenite phase (colored in red).

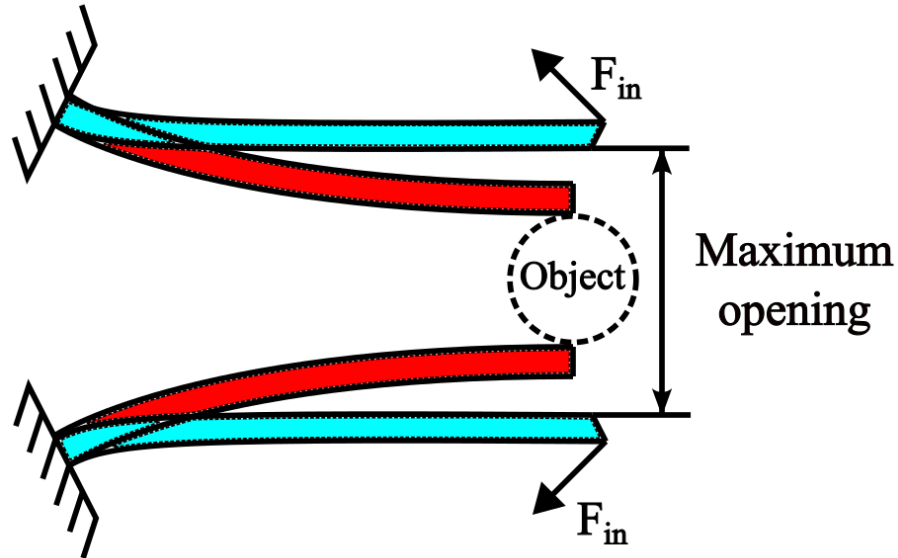


Figure 6-4- Total opening of the proposed gripper design.

The grasping force of the gripper is given by the triggering of the SME and can be calculated using the numerical model based on the experimental data of each actuator.

For this thesis project, the methodology requires a performance objective specification, specifically, the design must be focused on achieving a specific grasping force. The grasping force depends on the displacement of each beam. Creating a design based on the output force requires the analysis of the displacement of the beam and the output force of the beam. In the next sections two case studies will be analyzed.

6.3. CASE STUDY 1: AVERAGE OUTPUT FORCE.

To demonstrate the required steps to design a SMA-based gripper, a case study is analyzed, from the objective definition phase to the experimental results.

6.3.1. OBJECTIVE DEFINITION.

The performance objective definition should be the first step for the design of a SMA-based gripper. As mentioned in section 6.1, the gripper can be designed to satisfy a specific output force objective.

For this case study, the force performance objective is set to 3.6 N, as it is the average grasping force of the gripper designs shown in Table 6-2.

6.3.2. SHAPE MEMORY ALLOY NUMERICAL MODEL.

For this step of the methodology, two new samples of the shape memory alloy sheet used for beam-1 and beam-2 are obtained, the dimensions for the two new samples are the same as with beam-1 and beam-2, as a reminder, the length of each beam is 120 mm, where 20 mm are used to clamp the beam to have a 100 mm cantilever beam to analyze, the width is ~10 mm and the thickness is 5 mm. This is done to have two similar beams to be used as the gripper actuators, since the previous two samples were used for distinct tests and may behave differently from one another, this concern is based on the observed differences in elastic modulus of beam-1 and beam-2, for reference, read section 4.3.4.

Starting with two new samples cut from the same sheet should help to minimize the behavior differences of the two actuators. It is important to note that the data obtained from beam-1 and beam-2 is still useful to estimate some variables, for example, it is known from beam-2 that the maximum amount of force that can be delivered with the current dimensions of the SMA sheet is 5 N. Which is sufficient to obtain the specified grasping force objective.

The two new samples will be referred to as beam-3 and beam-4, respectively. Following the procedure from section 4.3, beam-3 is fixed as a cantilever beam and subject to an end load, ranging from 1 to 6 N.

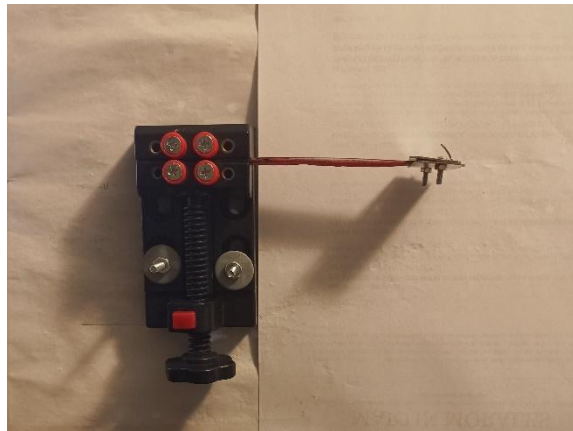


Figure 6-5.- Martensite phase analysis experimental setup.

As shown in Figure 6-5, beam-3 is fixed as a cantilever beam using a clamp, the remaining length of the beam is 100 mm, the same as with beam-1 and beam-2. At the end of the beam, two aluminum sheets are clamped together using two screws to support a hook to hold the load. The weight of the aluminum clamps is only 6 g, and their effect on the displacement of the beam can be ignored once the loads ranging from 100 to 600 g are applied. Using this experimental setup, a force-displacement curve can be plotted.

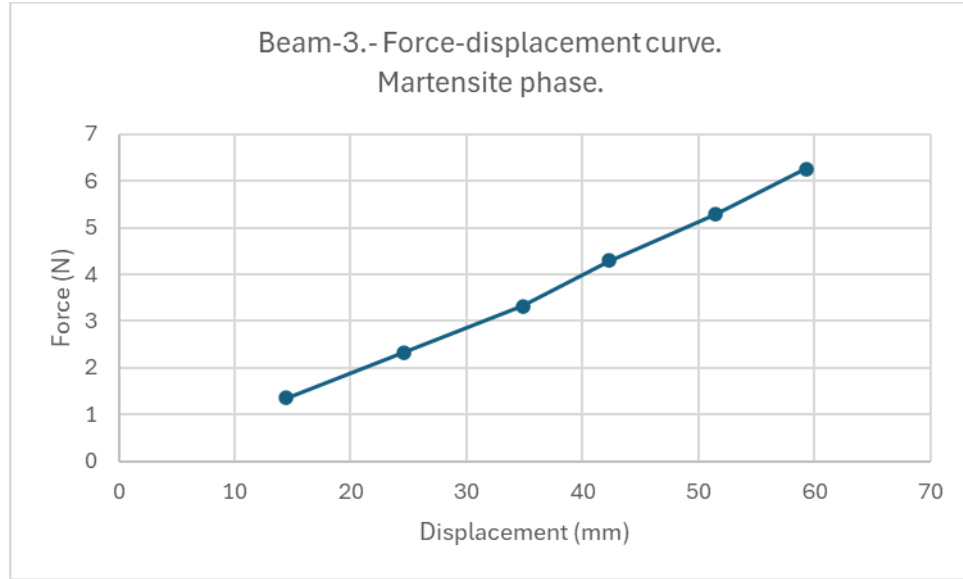


Figure 6-6.- Force-displacement curve for beam-3 in the martensite phase.

Using the force-displacement curve for beam-3, shown in Figure 6-6, The elastic slope is calculated as $k_m = 0.095$, and the martensite phase elastic modulus can be calculated using equation (4-4), this results in $E_m = 35$ GPa.

The calculated elastic modulus E_{m3} is then introduced into a numerical simulation model using ANSYS Workbench. The numerical simulation model requires the variables shown in Table 4-1. Considering that the new samples were cut from the same SMA sheet as beam-1 and beam-2, initially, the values for the poisson's ratio ν , the tangent modulus E_t , the elastic limit S_y , the temperature scaling parameter TSP , the maximum transformation strain MTS and the lode dependency parameter α are kept from the beam-2 numerical simulation analysis performed in section 4.3.4. The respective values for each material property are shown in Table 6-3, also, the austenite phase elastic modulus from beam-2 is also used, however, it does not have an effect since the simulation does not increase the temperature of the beam.

Property	Symbol	Value
Poisson's ratio	ν	0.3
Tangent modulus (GPa)	E_t	22
Elastic limit (MPa)	S_y	140
Temperature scaling parameter (MPa C ⁻¹)	TSP	1.43
Maximum transformation strain (mm mm ⁻¹)	MTS	0.061
Lode dependency parameter	α	0

Table 6-3.- Beam-3 base material properties for ANSYS numerical simulation model.

An initial numerical simulation is performed to test if the current model accurately represents the experimental results shown in Figure 6-6 using the mechanical properties shown in Table 6-3.

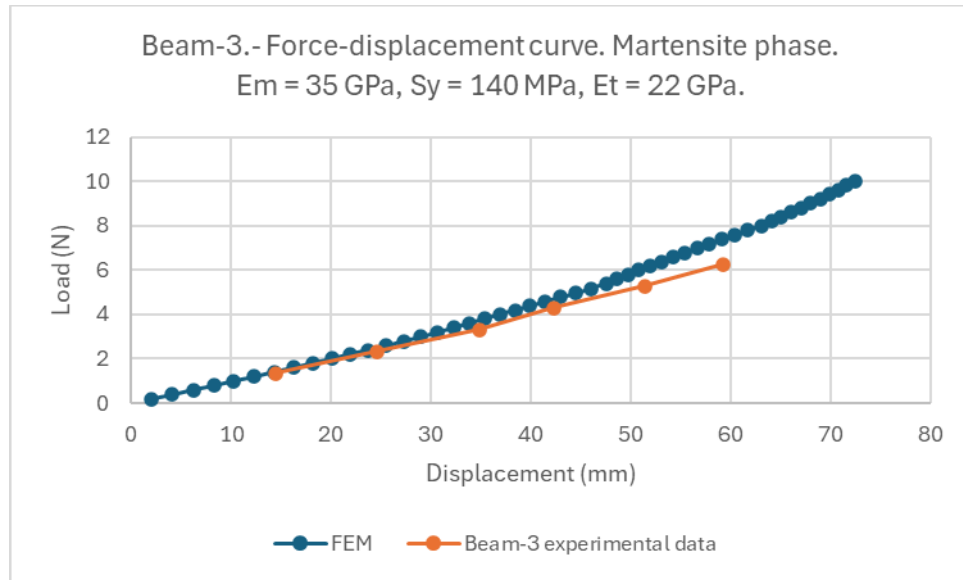


Figure 6-7.- Beam-3 numerical simulation results in the martensite phase using the material properties from Table 6-3.

As shown in Figure 6-7, the current numerical simulation model initially shows results similar to the experimental test, however, as the load increases the numerical model results diverge from the experimental test results. Based on the martensite phase sensitivity analysis performed in section 4.2.1, it is known that a decrease of the elastic limit affects the results by increasing the displacement of the beam in a high load range, without affecting the results in the lower ranges. Thus, two more numerical simulations were run, setting the elastic limit S_y to 70 MPa and 110 MPa, respectively.

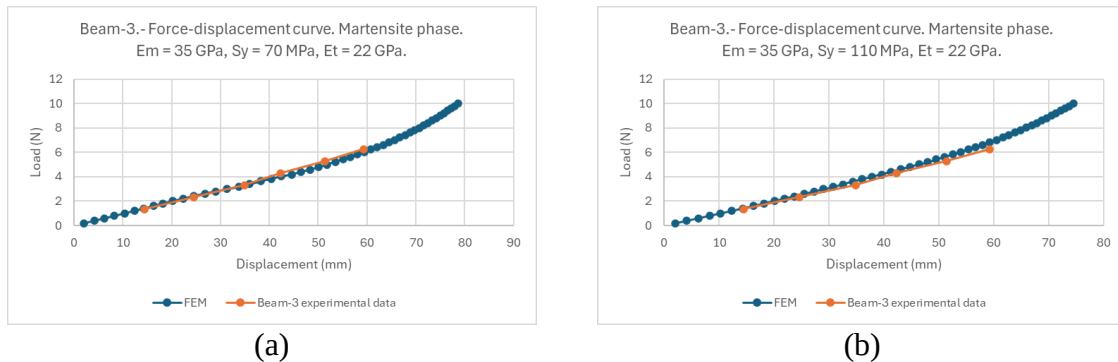


Figure 6-8.- Beam-3 Numerical simulation results in the martensite phase using an elastic limit S_y of (a) 70 MPa and (b) 110 MPa.

The numerical simulation results from Figure 6-8 are compared to the experimental results from Figure 6-6, see Table 6-4.

Experimental		Model $S_y = 70$ MPa	
Load (N)	d (mm)	FEM d(mm)	e%
1.35	14.44	14.22	1.52
2.33	24.60	25.43	3.37
3.32	34.88	35.75	2.49
4.30	42.33	44.95	6.19
5.28	51.44	53.17	3.36
6.26	59.29	60.32	1.74
		Average error:	3.11

(a)

Experimental		Model $S_y = 110$ MPa	
Load (N)	d (mm)	FEM d(mm)	e%
1.35	14.44	13.74	4.85
2.33	24.60	23.43	4.76
3.32	34.88	32.41	7.08
4.30	42.33	40.68	3.90
5.28	51.44	48.26	6.18
6.26	59.29	55.12	7.03
		Average error:	5.63

(b)

Table 6-4.- Numerical model results comparison to the experimental test results for force-displacement test for beam-3 in the martensite phase when using an elastic limit S_y of (a) 70 MPa and (b) 110 MPa.

As shown in Table 6-4, the calculated average error when using an elastic limit S_y of 110 MPa is 5.63%, with a maximum error of 7.08%, while the calculated average error when using an elastic limit of 70 MPa is only 3.11%, with a maximum error of 6.19%. The model shown in Figure 6-8.a is preferred and is used for the next steps of the methodology.

The next step in the methodology is to perform the cantilever beam experimental test in the austenite phase. To do this, a Ni-Cr sheet is used as a resistive material, fixed in place by two alligator clips connected to a KEATRONIC DC power supply, capable of delivering up to 30V/10A, the SMA is electrically isolated from the Ni-Cr resistance using masking tape.

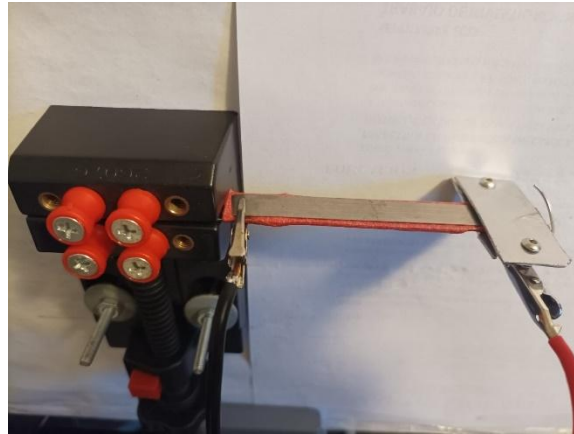


Figure 6-9.- Austenite phase analysis experimental setup.

Before adding the load and measuring the displacement, the power supply was set to deliver 9 A of current for 5 minutes, ensuring that the SMA beam is able to reach a temperature past $A_f = 60^\circ\text{C}$. The temperature of the beam is measured before adding the load using an MLX90614 IR temperature sensor connected to an Arduino, ensuring that the temperature of the beam is past A_f .

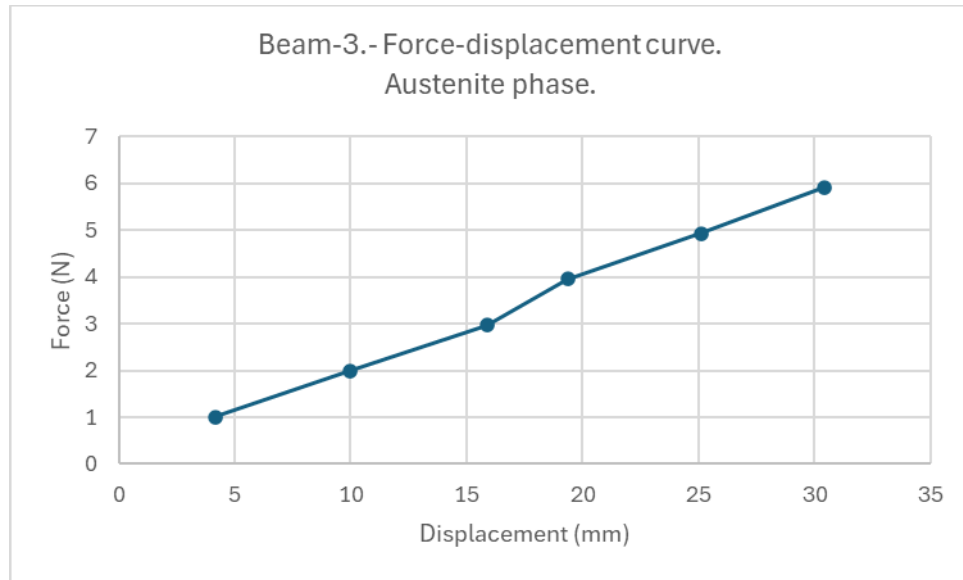


Figure 6-10.- Force-displacement curve for beam-3 in the austenite phase.

The resulting force-displacement curve from the previously described experimental setup is shown in Figure 6-10, using the gathered data, the elastic slope k is calculated as $k_a = 0.18$, and the austenite phase elastic modulus for beam-3 is calculated as $E_a = 70$ GPa using equation (4-4). The calculated austenite phase elastic modulus is then introduced to the material model, removing the beam-2 austenite phase elastic modulus, previously used as a placeholder.

A numerical simulation is run to analyze the behavior of the updated material model in the austenite phase. The resulting force-displacement curve is shown in Figure 6-11.

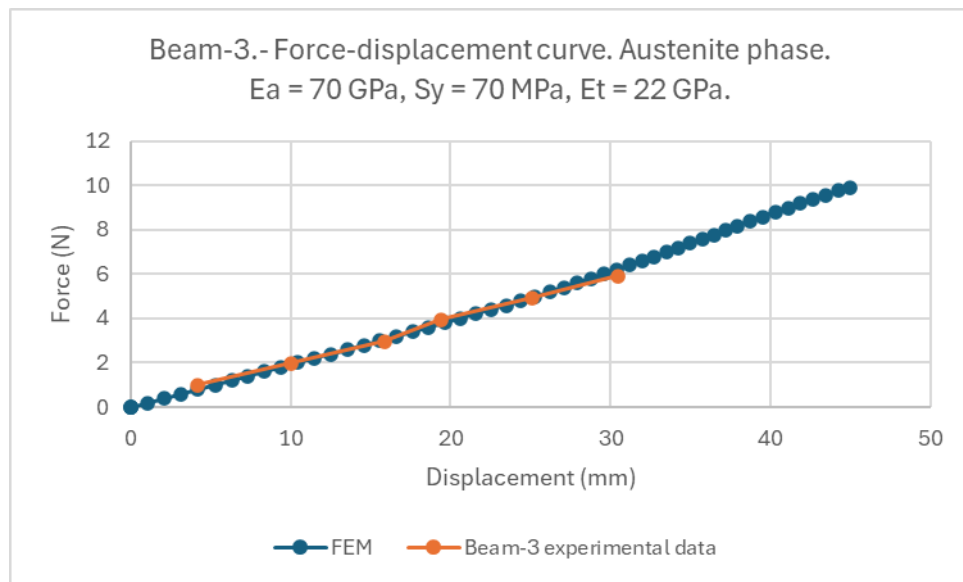


Figure 6-11.- Beam-3 numerical simulation results in the austenite phase.

The results of the numerical simulation model in the austenite phase are compared to the experimental results. As shown in Table 6-5, the numerical simulation model has an average error of 8.04% with a maximum error of 29.81%, this latter value is high, but it should be noted that the numbers being compared (4.16 and 5.40 mm) are very small. To put this into perspective, a 1 mm difference from 5 mm results in an error of 25%, on the other hand, a 1 mm difference from 30 mm results in an error of only 3%. It is also important to mention that a 1 N load is not likely to appear as an input force for the SMA actuators, as it is too low to create a phase transformation from twinned martensite to detwinned martensite to trigger the SME.

Experimental		Model	
Load (N)	d (mm)	FEM d(mm)	e%
1.01	4.16	5.40	29.81
1.99	9.99	10.56	5.71
2.97	15.87	15.50	2.33
3.95	19.37	20.23	4.44
4.93	25.12	24.75	1.47
5.92	30.46	29.10	4.46
Average error:			8.04

Table 6-5.- Numerical model results comparison to the experimental test results for force-displacement test for beam-3 in the austenite phase.

It should be noted that the process of obtaining the numerical simulation model for beam-3 was sped up due to the previous analysis results for beam-1 and beam-2. For reference of the entire process to estimate the elastic limit and tangent modulus of the beam, read section 4.3.

As mentioned before, two samples were cut in order to have two SMA actuators, the second sample will be referred to as beam-4. The procedure described above is performed a second time using beam-4 as the cantilever beam and is summarized below.

The experimental test is performed in the martensite phase with a load ranging from 100 to 600 g. The resulting force-displacement curve is shown in Figure 6-12.

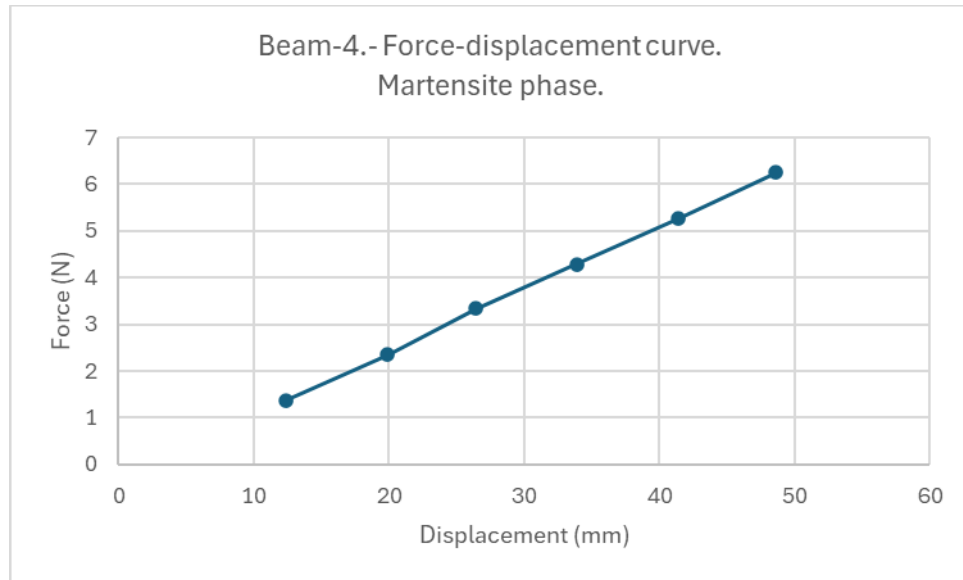


Figure 6-12.- Force-displacement curve for beam-4 in the martensite phase.

Using the information shown in Figure 6-12, the martensite phase elastic modulus E_m is calculated as $E_m = 34$ GPa. Afterwards, a numerical simulation is performed using the calculated elastic modulus.

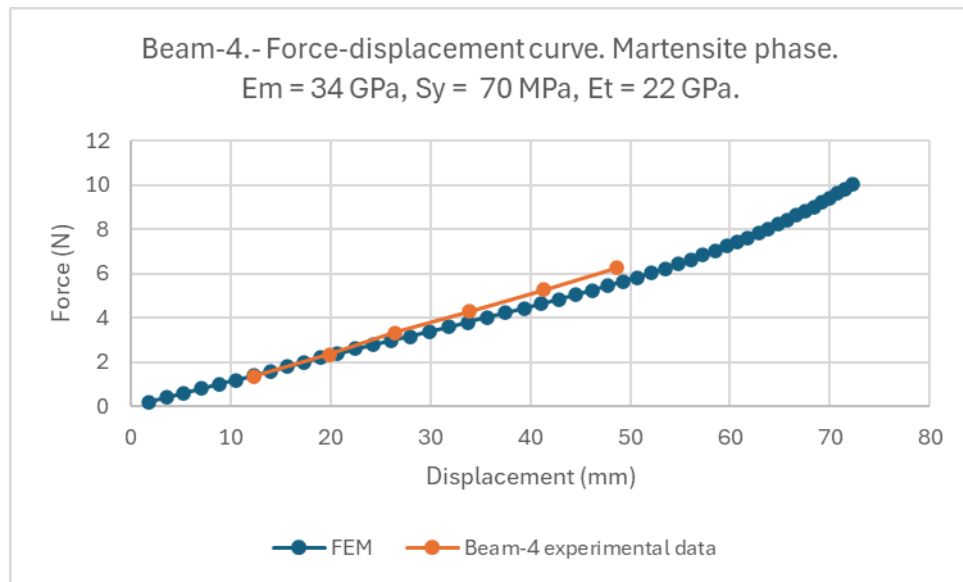


Figure 6-13.- Beam-4 numerical simulation results in the martensite phase using the elastic limit $S_y = 70$ MPa.

As shown in Figure 6-13, as the load increases the numerical simulation model and the experimental test results diverge, this is similar to what happened with beam-3, however, based on the sensitivity analysis performed in section 4.2.1, this time the elastic limit S_y should increase. A different numerical simulation is performed, this time, setting the elastic limit to 140 MPa, the same as with beam-1 and beam-2.

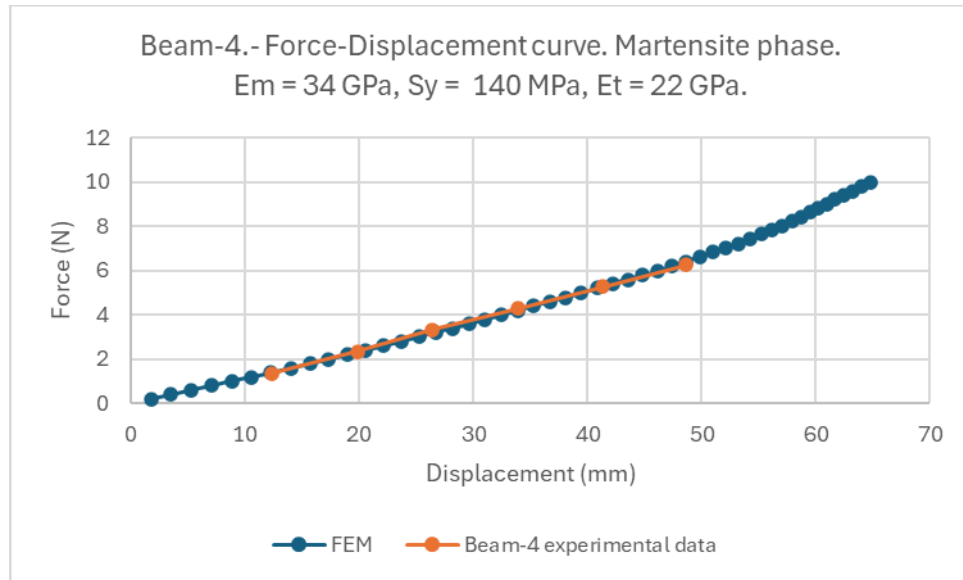


Figure 6-14.- Beam-4 numerical simulation results in the martensite phase using the elastic limit $S_y = 140$ MPa.

As shown in Figure 6-14, the numerical simulation model using the elastic limit $S_y = 140$ MPa better resembles the experimental test results.

Experimental		Model	
Load (N)	d (mm)	FEM d(mm)	e%
1.37	12.39	12.02	2.99
2.35	19.89	20.27	1.91
3.34	26.43	28.02	6.02
4.29	33.88	34.91	3.04
5.27	41.36	41.46	0.24
6.25	48.64	47.44	2.47
Average error:			2.78

Table 6-6.- Numerical model results comparison to the experimental test results for force-displacement test for beam-4 in the martensite phase.

As shown in Table 6-6, the calculated average error is 2.78% for the current model and the maximum error is 6.02%, these values are satisfactory for the purpose of this thesis project.

The next step is to obtain the force-displacement curve for beam-4 in the austenite phase at a temperature above A_f .

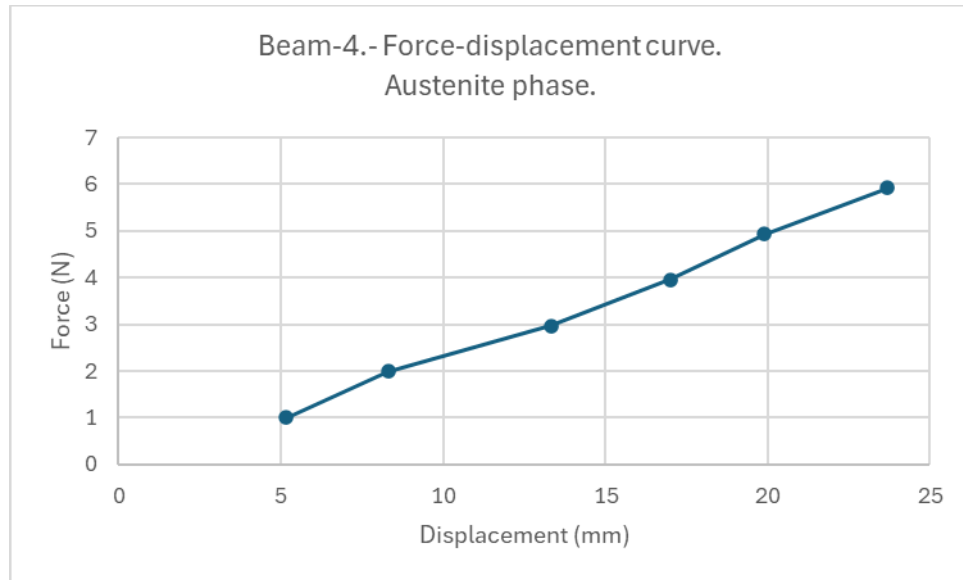


Figure 6-15.- Force-displacement curve for beam-4 in the austenite phase.

The force-displacement curve for beam-4 is shown in Figure 6-15 and the austenite phase elastic modulus is calculated as $E_a = 74$ GPa. Using the previous information, a numerical simulation is run in the austenite phase.

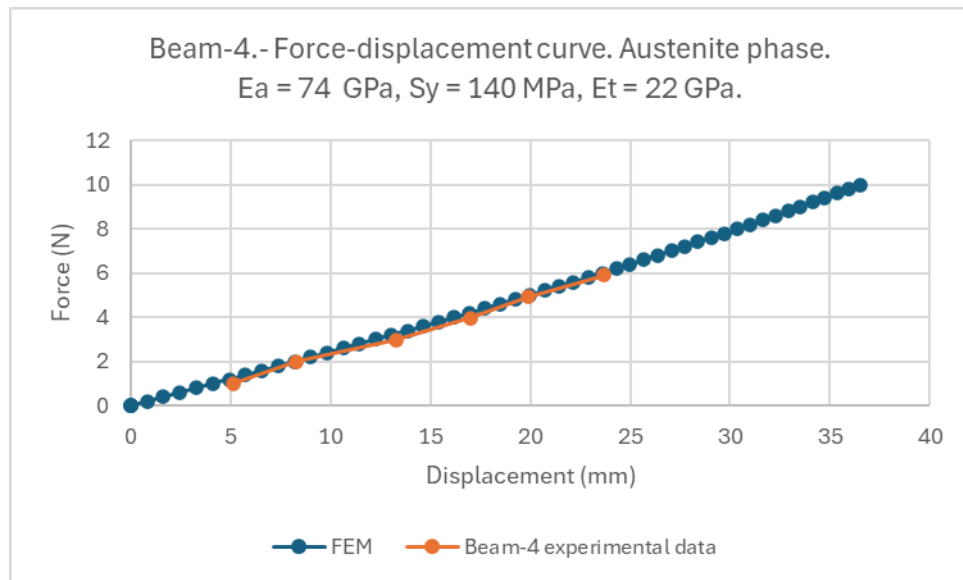


Figure 6-16.- Beam-4 numerical simulation results in the austenite phase.

As shown in Figure 6-16, the numerical simulation results resemble the experimental test results, a calculation of the error for each data point is performed and shown in Table 6-7.

Experimental		Model	
Load (N)	d (mm)	FEM d(mm)	e%
1.01	5.17	4.20	18.76
1.99	8.31	8.27	0.48
2.97	13.32	12.20	8.41
3.95	16.99	16.00	5.83
4.93	19.9	19.68	1.11
5.92	23.7	23.25	1.90
Average error:			6.08

Table 6-7.- Numerical model results comparison to the experimental test results for force-displacement test for beam-4 in the austenite phase.

The calculated average error is 6.08% and the maximum error is 18.76%, however, this error once again is present at the first data point, where a 1 mm deviation from the experimental value results in an error of about 25%. The calculated error values are satisfactory to continue with the methodology.

The numerical simulation model variable values for beam-3 and beam-4 are summarized in Table 6-8.

Property	Symbol	Beam-3	Beam-4
Young's modulus (Martensite) (GPa)	E_m	35	34
Young's modulus (Austenite) (GPa)	E_a	70	74
Poisson's ratio	ν	0.3	0.3
Tangent modulus (GPa)	E_t	22	22
Elastic limit (MPa)	S_y	70	140
Temperature scaling parameter (MPa $^{\circ}\text{C}^{-1}$)	TSP	1.43	1.43
Maximum transformation strain (mm mm^{-1})	MTS	0.061	0.061
Lode dependency parameter	α	0	0

Table 6-8.- Numerical simulation model mechanical properties for beam-3 and beam-4.

Shape memory alloys are known for their sensitivity to heat, which alters their mechanical properties. The two samples used for beam-3 and beam-4 were cut using an abrasive disk and while the cut was being made a stream of water was used to prevent some heat build-up. It is possible that the cuts still increased the temperature of the materials, affecting mainly the elastic limit S_y , however, most of the other variables are very similar.

6.3.3. MAXIMUM OUTPUT FORCE BASED ON NUMERICAL SIMULATION RESULTS.

As defined in section 6.3.1, the objective output force is set to 3.6 N. The numerical simulation model for beam-3 and beam-4 is used to estimate the maximum amount of output force that the actuators can deliver to ensure that the objective output force is achievable.

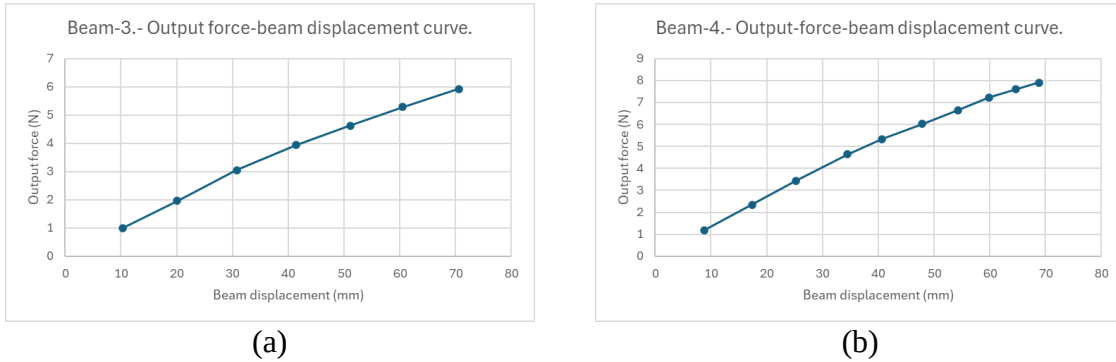


Figure 6-17.- Output force-beam displacement curve analysis for (a) Beam-3 and (b) Beam-4 using numerical simulation models.

As shown in Figure 6-17, beam-3 and beam-4 can deliver a maximum output force of 5.8 N and 7.8 N respectively. These output forces are sufficient to reach the specified output force performance objective of 3.6 N. If this were not the case, modifying the dimensions of the beams could help to increase the maximum output force of the actuators.

6.3.4. TWO-BEAM MODEL APPROXIMATION.

In order to execute the optimization program to calculate the required input force to deliver the objective output force, a two-beam model for beam-3 and beam-4 needs to be found, since the optimization algorithm in MATLAB cannot consider the elastoplastic behavior of the SMA material. As shown in section 5.2, the two-beam model approximation consists of finding the appropriate length of two beams in series, the beam connected to the fixed support has an elastic modulus equal to the tangent modulus E_t of the SMA material model and the beam with the free end has an elastic modulus equal to the martensite phase elastic modulus of the SMA material model (Figure 5-9).

Based on Figure 6-17.a, an output force of 3.6 N requires a beam tip displacement of $T_d = 37.64$ mm. For a cantilever beam with a vertical load, various two-beam model simulations were run, varying the length ratio of the two sections of the two-beam model from 0:100 to 40:60, where the first value is the length ratio with the tangent modulus section and the second value is the length ratio with the martensite phase elastic modulus section.

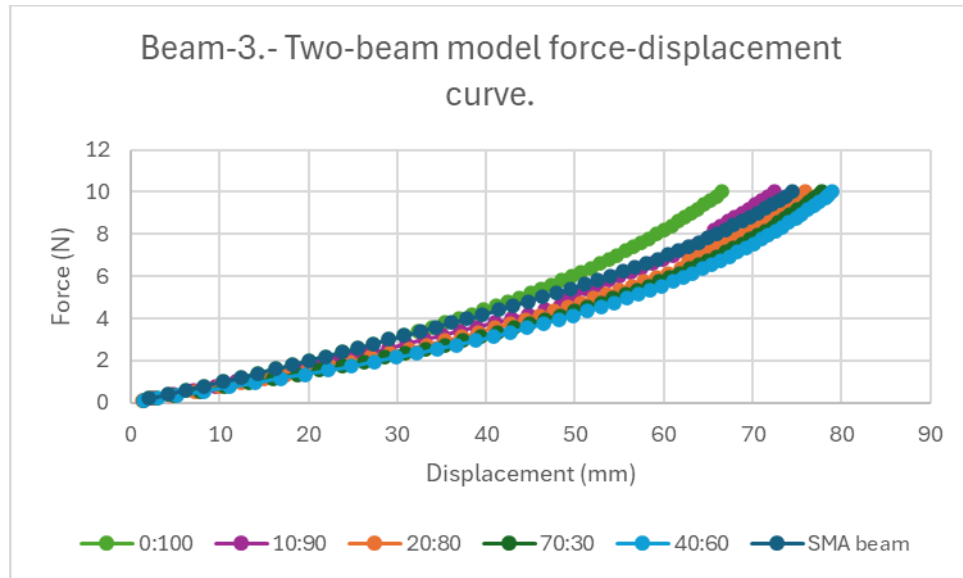


Figure 6-18.- Beam-3 force-displacement curve of the SMA behavior compared to the two-beam model using different ratios.

As shown in Figure 6-18, the two-beam model ratios for 20:80, 70:30 and 40:60 differ from the SMA numerical simulation model behavior. Figure 6-19 shows a zoom in the displacement range of interest comparing the force-displacement curve for the SMA beam model and the two remaining two-beam models.

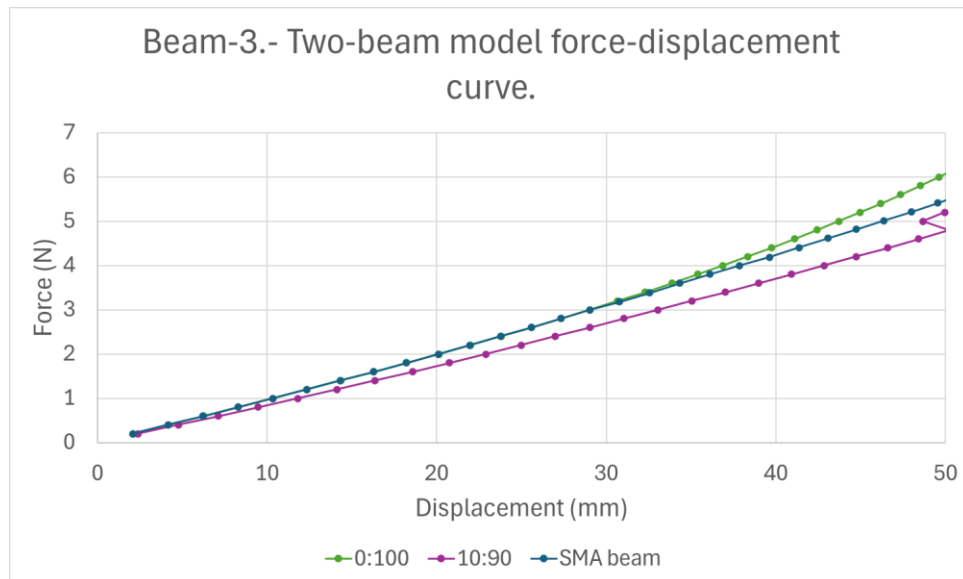


Figure 6-19.- Beam-3 force-displacement curve of the SMA behavior compared to the two-beam model using a ratio of 0:100 and 10:90.

As shown in Figure 6-19, the two-beam model with a ratio of 0:100 better resembles the behavior of the SMA numerical model for the specified range of 30 to 40 mm.

A similar analysis is performed for beam-4, where the required displacement to deliver an output force of 3.6 N is $T_d = 26.23$ mm, based on Figure 6-17.b. The results are similar to beam-3, as shown in Figure 6-20, the two-beam model with a ratio of 0:100 better represents the behavior of the SMA material model.

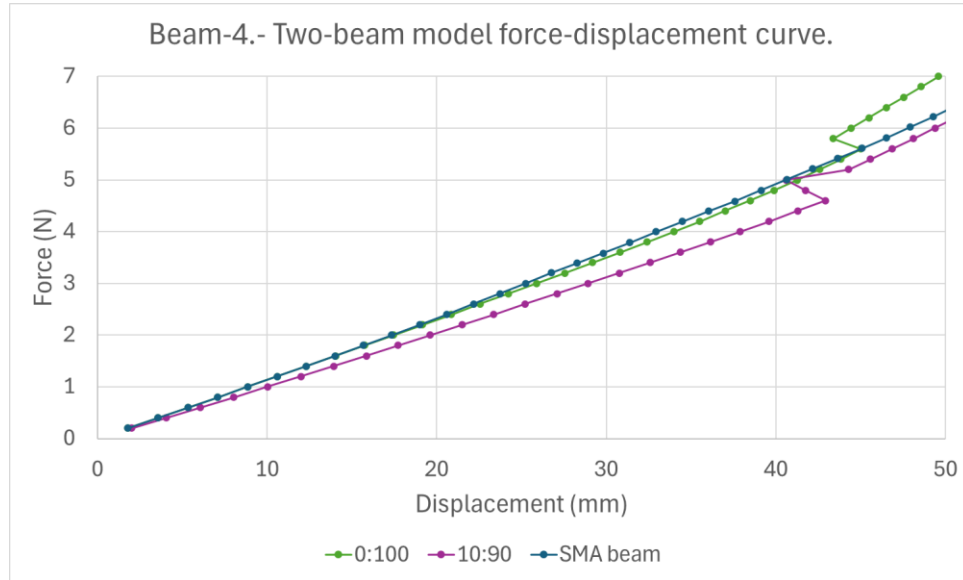


Figure 6-20.- Beam-4 force-displacement curve of the SMA behavior compared to the two-beam model using a ratio of 0:100 and 10:90.

Currently, the input load direction is unknown. The results shown in Figure 6-18, Figure 6-19 and Figure 6-20 were calculated with a horizontal beam ($\eta = 0^\circ$) and a vertical load ($\varphi = 270^\circ$). However, the precision of the two-beam model may be affected by the direction of the input force relative to the beam. To have more confidence in the selected two-beam model, more numerical simulations were run, where the input force angle φ varies from 200 to 260 degrees. The input load F_{in} increases from 0 to 10 N in each simulation to generate the force-displacement curve and the beam rotation η is set to 0° for all simulations, since only the direction of the force relative to the beam is important. After obtaining the numerical simulation results, a second order polynomial equation is used to calculate the input force value of the model based on the required total displacement of the beam. The input force is calculated based on the total displacement since currently only the required total displacement of the beams are known. The results of this analysis are summarized in Table 6-9 and Table 6-10.

T_d (mm)	φ (deg)	Force (N)				Error (%)		
		SMA	0:100	10:90	20:80	0:100	10:90	20:80
37.64	260.00	3.57	3.76	3.12	2.86	5.53	12.49	19.80
37.64	250.00	3.39	3.61	3.01	2.75	6.34	11.23	19.07
37.64	240.00	3.28	3.59	3.02	2.72	9.43	8.12	17.18
37.64	230.00	3.36	3.73	3.10	2.79	10.84	7.79	16.99
37.64	220.00	3.57	3.94	3.35	2.97	10.33	6.19	16.86
37.64	210.00	3.90	4.29	3.72	3.28	9.78	4.70	16.01
37.64	200.00	4.52	5.06	4.33	3.80	12.03	4.22	15.82
Average:						9.18	7.82	17.39

Table 6-9.- Beam-3 two-beam model numerical simulation results for case study 1.

T_d (mm)	φ (deg)	Force (N)				Error (%)		
		SMA	0:100	10:90	20:80	0:100	10:90	20:80
26.23	260	2.86	2.85	2.39	2.25	0.43	16.67	21.41
26.23	250	2.81	2.85	2.39	2.24	1.40	14.87	20.12
26.23	240	2.84	2.93	2.49	2.31	3.34	12.22	18.83
26.23	230	2.95	3.10	2.70	2.42	5.10	8.67	18.21
26.23	220	3.18	3.37	3.05	2.63	5.93	4.30	17.43
26.23	210	3.62	3.83	3.48	2.99	5.73	3.95	17.27
26.23	200	4.27	4.65	4.08	3.65	8.86	4.29	14.49
Average:						4.40	9.28	18.25

Table 6-10.- Beam-4 two-beam model numerical simulation results for case study 1.

As shown in Table 6-9, for beam-3, the two-beam model with a ratio of 10:90 is better for most cases, with an average absolute percentage error of 7.82% when compared to the SMA model. This demonstrates that the initial selection of the two-beam model with a ratio of 0:100 is not the best for all input load directions, in reality, it can be observed in Table 6-9 that the average error for this model increases as the input load angle decreases. The opposite happens with the two-beam model with a ratio of 10:90, where the error decreases as the input load angle decreases.

As shown in Table 6-10, for beam-4, the two-beam model with a ratio of 0:100 has an average error of 4.40%, in this case, the initial selection was correct, the two-beam model with a ratio of 0:100 shows good results with the different input load angles analyzed here.

6.3.5. OPTIMIZATION PROBLEM FORMULATION AND SOLUTION.

The next step in the methodology for the design of a SMA-based compliant gripper is to use the optimization algorithm shown in section 5.4 to calculate the required input force F_{in} , input force angle φ and SMA actuator rotation η to ensure that the displacement of the beam is sufficient to deliver the pre-defined objective output force of 3.6 N.

The objective total tip displacement T_d depends on the required output force and can be calculated using the numerical simulation results shown in Figure 6-17. For this case study, for beam-3, an output force of 3.6 N can be obtained with a tip displacement T_d of $T_d = 37.64$ mm, calculated using a second order polynomial regression of the data shown in Figure 6-17.a. Using the same method, the required total displacement T_d for beam-4 to deliver an output force of 3.6 N is calculated as $T_d = 26.23$ mm. The previously mentioned displacements become the total required displacements T_d introduced to the objective function fun .

The optimization problem is formulated as follows:

Minimize:

$$fun = \left| \frac{T_{di} - T_d}{T_d} \right| * 100 + \left| \frac{\theta_i - \theta_{tip}}{\theta_{tip}} \right| * 100$$

Subj. to:

$$\begin{aligned} F_{min} &\leq F_{in} \leq F_{max} \\ 202.5 &\leq \varphi \leq 270 \\ 22.5 &\leq \eta \leq 67.5 \end{aligned}$$

Where T_d is the total displacement objective of the beam, T_{di} refers to the displacement at the current iteration of the optimization algorithm, θ_{tip} refers to the tip angle objective, which should be set to 1° to avoid non linearities in the objective function fun .

The optimization variables F_{in} , φ and η are bounded based on the required total displacement T_d . For beam-3, it is known that it can be achieved with a force of about 4 N, based on the numerical simulation results shown in Figure 6-8.a, however, these results were calculated using a horizontal beam and a vertical load. To ensure that the optimization algorithm can modify the rotation of the beam, the input force bounds are calculated using the procedure shown in section 5.3. This results in an input force lower bound of $F_{min} = 2$ N, and an input force upper bound of $F_{max} = 6$ N. Using the same method for beam-4, considering that the required total displacement T_d can be achieved with a load of about 3 N based on the numerical simulation results shown in Figure 6-14, the input force lower and upper bounds are calculated as $F_{min} = 1.5$ N and $F_{max} = 4.5$ N. For both cases, the specified bounds for the input load angle φ and the rotation of the beam η are kept from section 5.3.

The optimization algorithm *ga* was used since there is not a starting point for *fmincon* to work with. The optimization problem solutions for each beam required displacements are shown in Table 6-11.

Actuator	Input force F_{in} (N)	Input force angle φ (deg)	Beam rotation η (deg)
Beam-3	4.26	240.64	32.65
Beam-4	4.42	232.62	26.35

Table 6-11.- Optimization algorithm results for beam-3 and beam-4.

With the results from Table 6-11, two numerical simulations were run incorporating the specified beam rotation η , as well as the input force F_{in} and input force angle φ , see Figure 6-21.

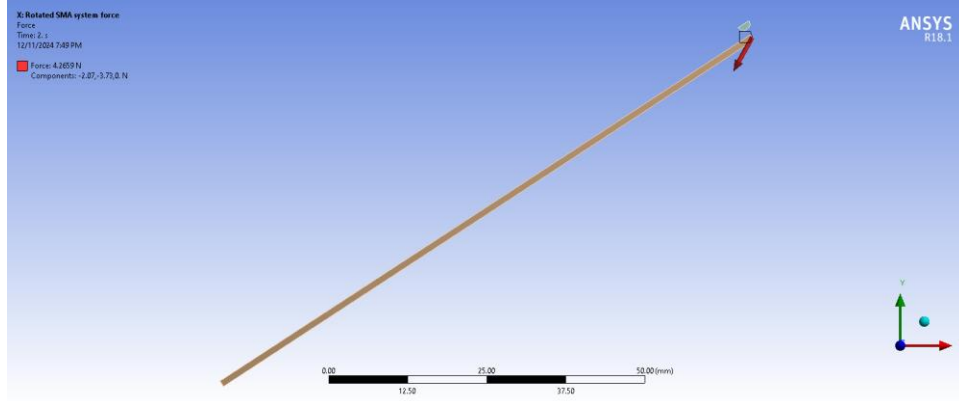


Figure 6-21.- Beam-3 numerical simulation set up.

As shown in Figure 6-21, the beam is modeled with the rotation specified by the optimization algorithm, in this case, $\eta = 32.65^\circ$, and the force applied to the beam (red arrow) has a magnitude of $F_{in} = 4.26$ N and a direction of $\varphi = 240.64^\circ$, relative to the horizontal axis.

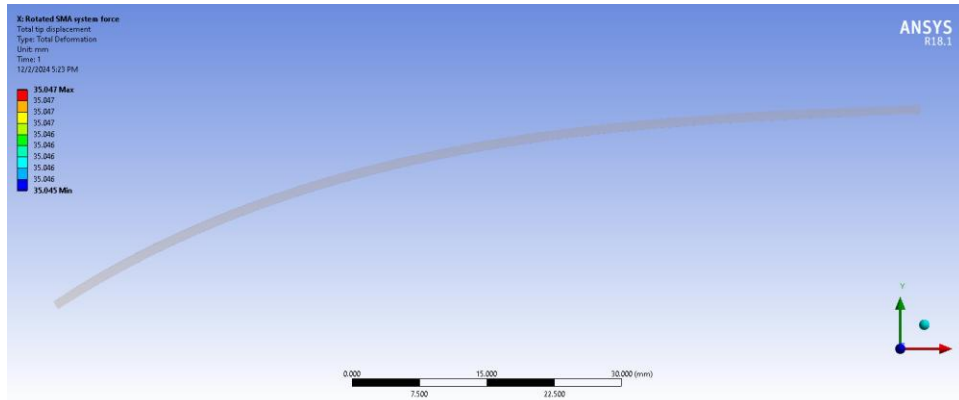


Figure 6-22.- Beam-3 tip displacement using optimization algorithm results.

As shown in Figure 6-22, beam-3 is displaced by 35.04 mm using the optimization algorithm results, which is close to the required displacement of 37.64 mm, with an absolute percentage error of 6.9%, calculated using equation (6-1), where $x_{expected}$ is the expected result and x represents the actual numerical simulation result.

$$\%e = \left| \frac{x_{expected} - x}{x_{expected}} \right| * 100 \quad (6-1)$$

The maximum strain should also be considered before moving on. The material is a Ni-Ti SMA and the maximum recoverable strain is 8%. With the previously specified loading conditions, the maximum equivalent elastic strain is calculated using the same numerical

simulation as 0.4%. This value is not concerning since it is well-below the limit of 8% and next, the output force can be calculated.

The numerical simulation result for the output force is 3.42 N. This is expected since the displacement of the beam was below the target displacement and the output force is proportional to the displacement of the beam, the calculated absolute percentage error is 5.26% in this case. As shown in Figure 6-23, the reaction force on the obstruction is normal to the surface of the beam, this is the expected behavior and the reason why it is important for the optimization algorithm to rotate the beam, to ensure that the force provided by the actuator is directed towards the grasped object, as shown in Figure 5-5.

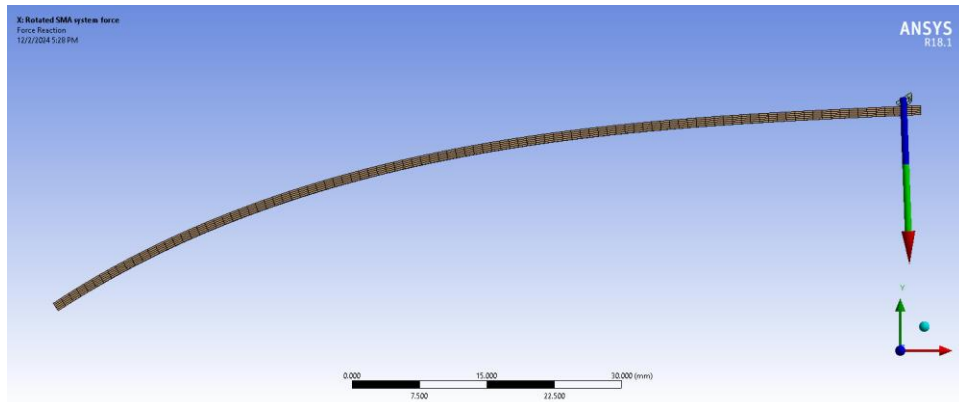


Figure 6-23.- Beam-3 output force reaction direction.

The numerical simulation displacement results for beam-4 are shown in Figure 6-24. As shown, the displacement is 27.69 mm, while the required displacement is 26.3 mm, resulting in an absolute percentage error of 5.31%.

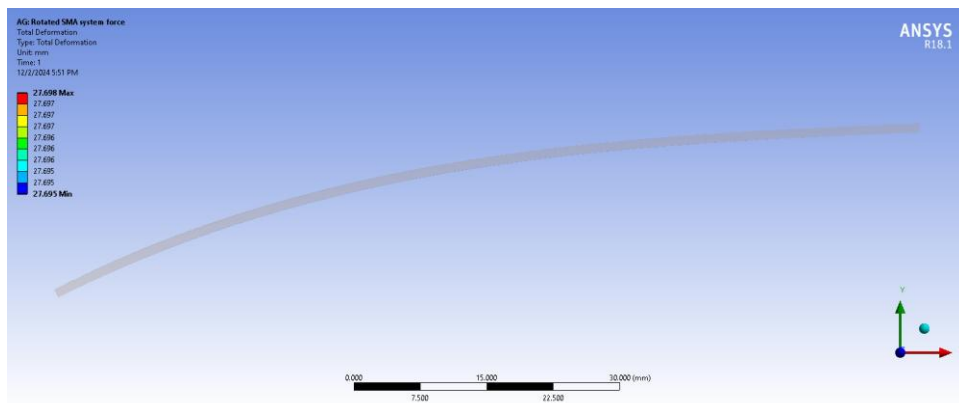


Figure 6-24.- Beam-4 tip displacement using optimization algorithm results.

For this case, the maximum equivalent elastic strain is calculated as 0.3%, which is below the maximum recoverable strain of 8%.

The resulting output force for this case is 3.78 N, slightly above the objective output force of 3.6 N, resulting in an absolute percentage error of 4.7%. Similarly, the rotation of

the beam allows the actuator to direct the grasping force in the correct direction, as shown in Figure 6-25.

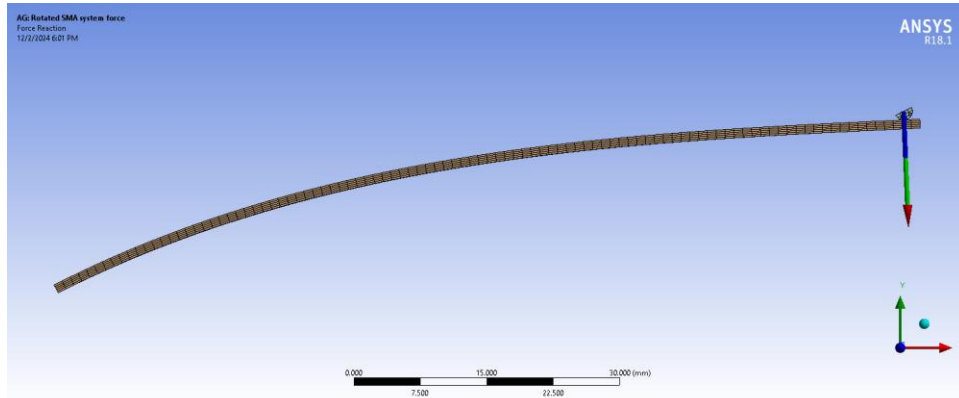


Figure 6-25.- Beam-4 output force reaction direction.

Up to this point, a force has been used in the numerical simulations to ensure that the calculations are correct, however, to reproduce the results experimentally, a method for applying the mentioned force has to be found.

6.3.6. BIAS SPRING ANALYSIS.

In order to deliver the input force to the tip of the beam, a bias spring can be utilized to deliver a force in a specific direction. The implementation of a bias spring only requires the use of the well-known Hooke's Law to calculate the necessary length (x) that the spring has to extend to deliver a force F_s , based on the spring constant k_s .

$$F_s = k_s x \quad (6-2)$$

In order to replace the input force, several calculations should be performed. Consider the rotated beam shown in Figure 6-26. In the starting position, the beam is undeflected and the bias spring has an initial length L_{s1} . In this initial position, the bias spring is not applying any force and is parallel to the beam. Point A_1 and point B_1 represent the tip of the beam where the bias spring is connected and the base of the spring, respectively.

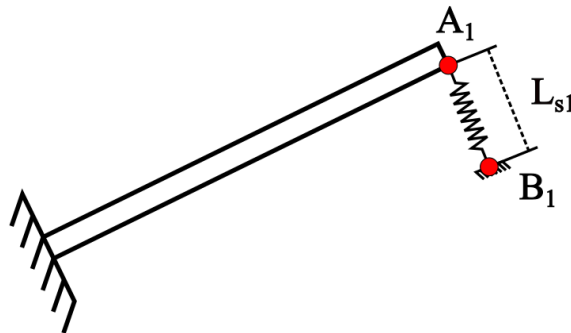


Figure 6-26.- Initial position of undeflected beam connected to a bias spring.

Using the optimization algorithm results from the input force analysis for each beam (F_{in} , φ and η), it is possible to calculate the required displacement of the spring base by first calculating the required spring length L_{s2} based on the required input force F_{in} , as shown in equation (6-3).

$$L_{s2} = L_{s1} + \frac{F_{in}}{k_s} \quad (6-3)$$

After calculating the length of the spring required to produce the input force F_{in} , the spring base position B_2 (Figure 6-27.a) needs to be calculated by adding the required spring length L_{s2} to the displaced beam tip position A_2 (Figure 6-27.b), which is known from the numerical simulation results where the input force is used.

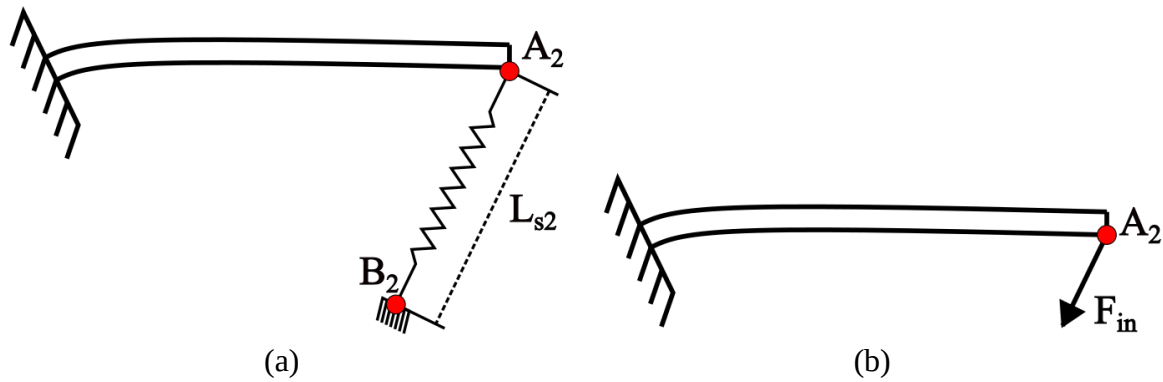


Figure 6-27.- Beam displacement using: (a) bias spring and (b) input force.

The position of point B_2 can be calculated using equations (6-4) and (6-5), where the x and y coordinates of point B_2 are calculated by adding the horizontal and vertical components of the required spring length L_{s2} to the deflected beam tip position A_2 based on the required input load angle φ .

$$B_{2x} = A_{2x} + L_{s2} \cos(\varphi) \quad (6-4)$$

$$B_{2y} = A_{2y} + L_{s2} \sin(\varphi) \quad (6-5)$$

After calculating the coordinates for point B_2 , the required spring base displacement can be calculated by subtracting the required spring base position B_2 coordinates to the initial spring base position coordinates B_1 , as shown in equations (6-6) and (6-7).

$$\text{Horizontal spring base displacement} = B_{2x} - B_{1x} \quad (6-6)$$

$$\text{Vertical spring base displacement} = B_{2y} - B_{1y} \quad (6-7)$$

The spring used in this thesis project has a spring constant of $k_s = 0.087$. A numerical simulation is set up for beam-3 and beam-4, replacing the input force by the spring, which is connected from one end to the beam and to a mobile base on the other end, as shown in Figure 6-28.

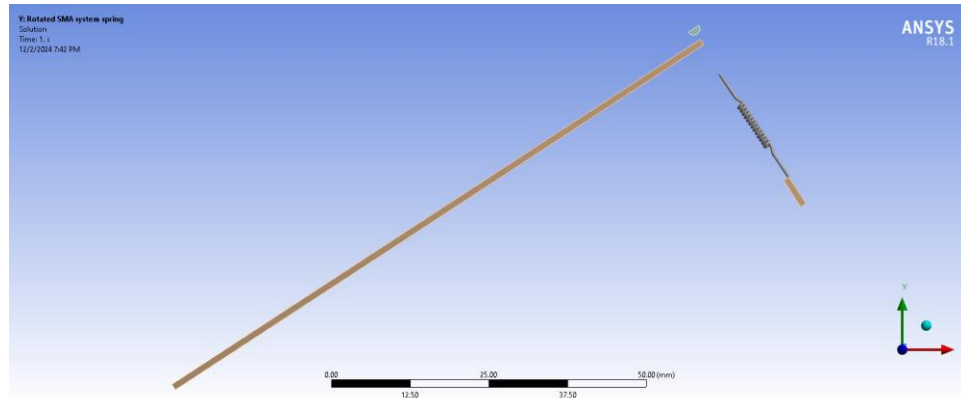
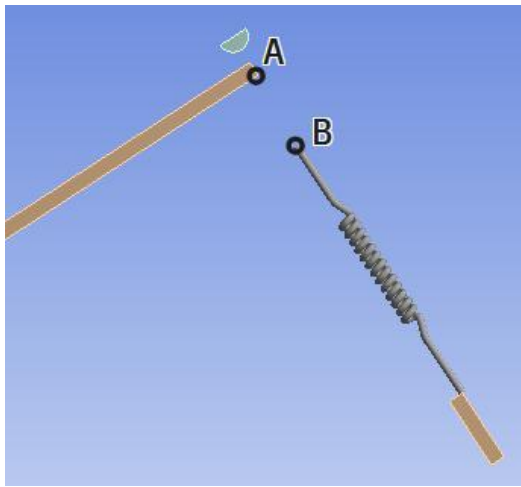


Figure 6-28.- Numerical simulation model for beam-3 where the force is given by a tension spring connected to a mobile base.

The distance spanning from the tip of the beam (point A) to the spring connection B (Figure 6-29.a) has a length of 5.5 mm, and it represents the size of a connection ring for the spring (Figure 6-29.b). The other end does not require this distance since the reference point is set at the base of the spring, not including the ring. This distance is defined as a remote attachment in ANSYS Workbench, and it still represents a connection between points A and B.



(a)



(b)

Figure 6-29.- (a) Remote attachment from the tip of the beam to the spring and (b) spring connection ring.

The geometric analysis that results in equations (6-4) to (6-7) is performed to calculate the required position of the mobile base (point B in Figure 6-27.a), this time considering the spring connection ring, and the simulation setup is run for beam-3 and beam-4.

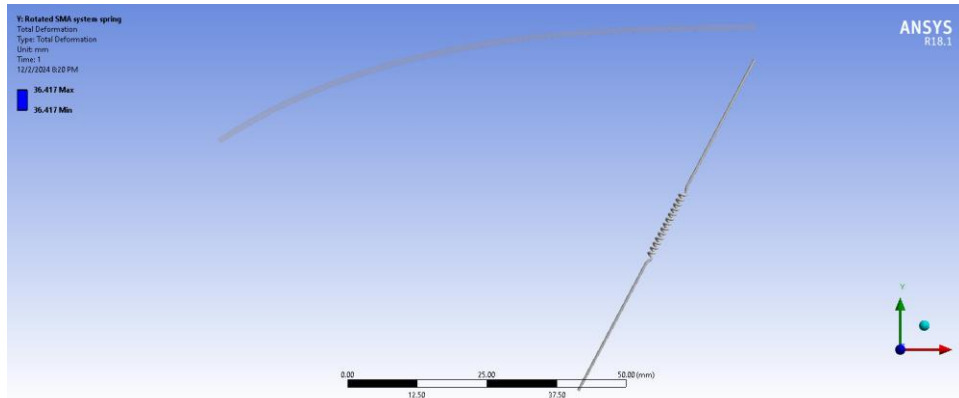


Figure 6-30.- Beam-3 numerical simulation results implementing a bias spring as an input force.

As shown in Figure 6-30, the numerical simulation results show that the displacement can also be achieved using a bias spring.

6.3.7. EXPERIMENTAL RESULTS.

In this section, an experimental test is shown based on the results from sections 6.3.1 to 6.3.6. This experimental analysis will demonstrate that the numerical simulation model and the optimization algorithm results are accurate and reproducible using the physical SMA actuators beam-3 and beam-4.

Starting with beam-3, a clamp was used to hold the SMA actuator, representing the fixed end of the beam. A Ni-Cr sheet was positioned on top of the SMA which is covered by a masking tape to prevent current from flowing through the Ni-Ti alloy. At the free end, an aluminum clamp was positioned to hold the Ni-Cr sheet and a small metal hook to connect the spring. The power supply cables were connected close to the fixed end and to the aluminum clamp to deliver power to the Ni-Cr sheet to trigger the SME. This experimental setup is shown in Figure 6-31, and it works for both beam-3 and beam-4.

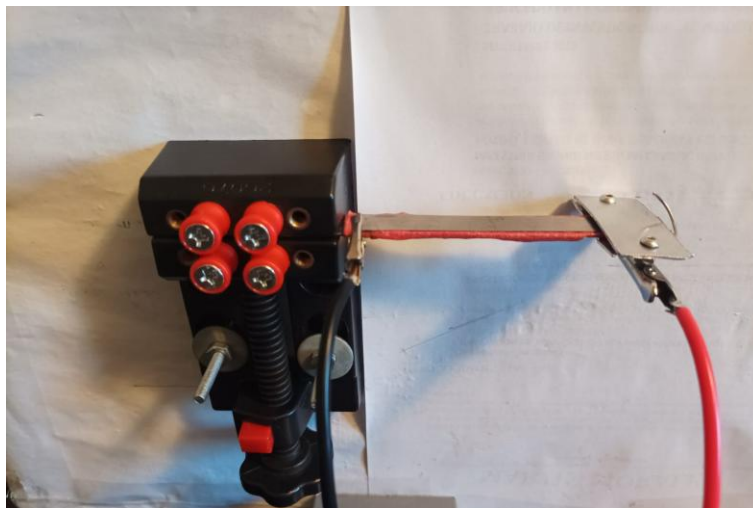


Figure 6-31.- Experimental setup for beam-3 and beam-4.

The experimental test follows the procedure described below:

1. The actuator is set up as shown in Figure 6-31. There is no load, and the power supply is off.
2. The spring is connected to the free end of the beam using the hook and the base of the spring is moved to the required position to deliver the required force to obtain the necessary displacement.
3. The spring base is fixed in place, and the power supply is turned on. A digital force gauge model ZMF-500 is used to measure the output force of the actuator. A KEATRONIC 30V/10A DC Power Supply was used to trigger the SME, where the power supply delivers a 9 A current through the Ni-Cr resistance to increase the temperature of the actuators.

It is important to mention that, in order to simplify the experimental setup, the beam rotation η is not considered. To ensure that the results are not affected, the beam rotation η is subtracted from input force angle φ to account for the rotation of the beam. To exemplify this, consider the cantilever beam shown in Figure 6-32.a, the rotation of the beam η is 30 degrees and the input force angle φ is 245 degrees. To do an experimental analysis of this, the beam is clamped in a horizontal position, meaning that the system has to rotate by 30 degrees in the clockwise direction. The input force angle is also rotated by 30 degrees and the new input force angle is 215 degrees, as shown in Figure 6-32.b. This rotation does not affect the input force F_{in} .

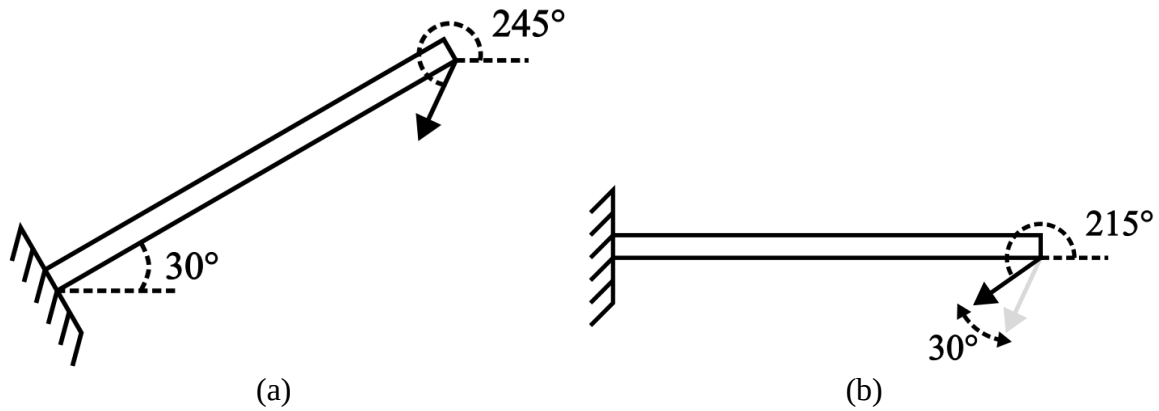


Figure 6-32.- Example of system rotation: (a) original setup and (b) rotated system with the same input force direction relative to the beam.

A bias spring with a spring constant $k_s = 0.087$ is used to reproduce the results shown in Figure 6-22 and Figure 6-23 for beam-3. Using equation (6-2), the distance that the spring needs to travel is calculated as $x = 48.9$ mm to deliver the required input force of $F_{in} = 4.26$ N. Considering that the initial length of the spring is $L_{s1} = 19.2$ mm, the spring should have a length of $L_{s2} = 68.1$ mm to deliver the required input force. The required input force angle φ is calculated as 207.9 degrees to consider the rotation of the beam by subtracting the beam rotation $\eta = 32.65^\circ$ to the input force angle $\varphi = 240.64^\circ$.

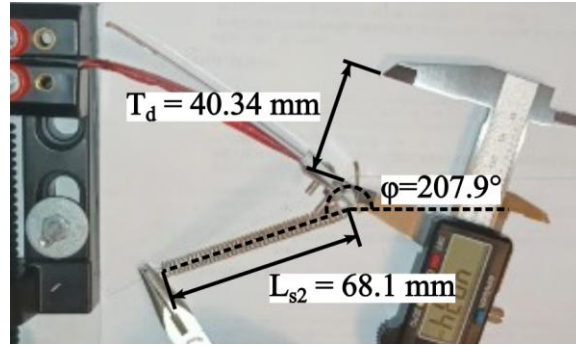


Figure 6-33.- Beam-3 experimental test displacement using a bias spring.

As shown in Figure 6-33, using the optimization algorithm results, the beam is displaced by a magnitude of $T_{dex} = 40.34$ mm. In section 6.3.5, the required displacement to achieve an output force $F_{out} = 3.6$ N was calculated as $T_{dreq} = 37.64$ mm, this means that the experimental displacement T_{dex} has a 7.17% absolute percentage error when compared to the required displacement T_{dreq} .

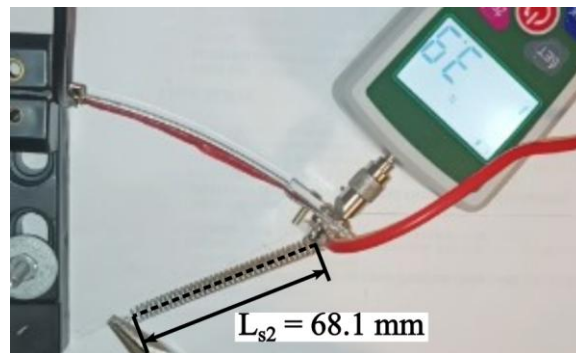


Figure 6-34.- Beam-3 experimental test output force using a bias spring.

As shown in Figure 6-34, the beam is able to deliver an output force of $F_{out} = 3.9$ N. The output force has an absolute percentage error of 8.33% when compared to the objective output force of $F_{out} = 3.6$ N.

The same procedure was performed to corroborate the numerical simulation results shown in Figure 6-24 and Figure 6-25 for beam-4. In this case, the spring needs to travel a distance of $x = 50.8$ mm to deliver the required input force of $F_{in} = 4.42$ N, resulting in a final length of $L_{s2} = 70$ mm. The input force angle is calculated as 206.2 degrees by subtracting the beam rotation $\eta = 26.35^\circ$ to the input force angle $\phi = 232.62^\circ$.

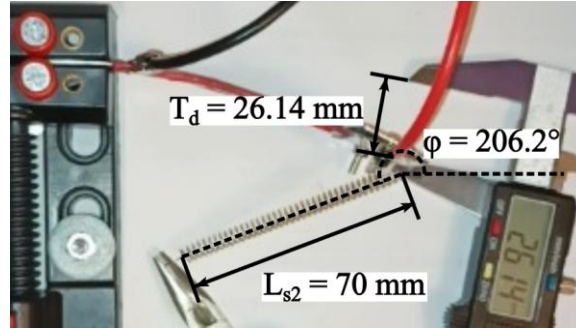


Figure 6-35.- Beam-4 experimental test displacement using a bias spring.

As shown in Figure 6-35, using the optimization algorithm results, the beam is displaced by a magnitude of $T_{dex} = 26.14$ mm. In section 6.3.5, the required beam displacement to deliver an output force $F_{out} = 3.6$ N is $T_{dreq} = 26.3$ mm, based on this, the experimental displacement T_{dex} has an absolute percentage error of 0.60% when compared to the required displacement T_{dreq} .

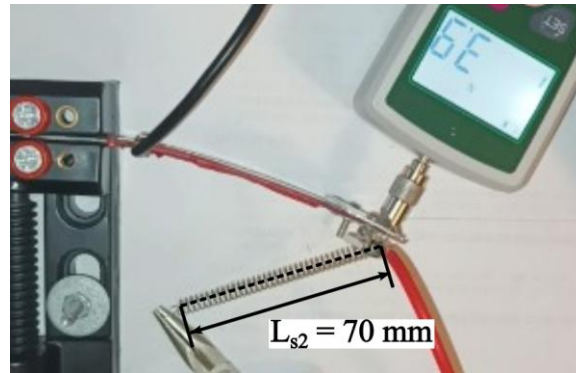


Figure 6-36.- Beam-4 experimental test output force using a bias spring.

As shown in Figure 6-36, the beam is capable of delivering an output force of $F_{out} = 3.9$ N to the force gauge. This results in an absolute percentage error of 8.33% compared to the objective output force of $F_{out} = 3.6$ N.

Both actuators are able to deliver a force of 3.9 N, which demonstrates that the methodology was properly used to successfully attain the objective output force of 3.6 N, in both cases, exceeding the objective output force by 0.3 N. It was also demonstrated that the numerical simulation models for beam-3 and beam-4 were accurate. The next step is to combine the numerical simulations for beam-3 and beam-4 to represent the behavior of the gripper.

6.3.8. CASE STUDY 1 GRIPPER DESIGN NUMERICAL SIMULATION.

In this section, the numerical simulation results for beam-3 and beam-4 are combined into a single numerical simulation to demonstrate the performance of the gripper design. In addition, the obstructions used in both simulations are replaced by a single object with a diameter of 50 mm that is desired to be held with a grasping force of 3.6 N.

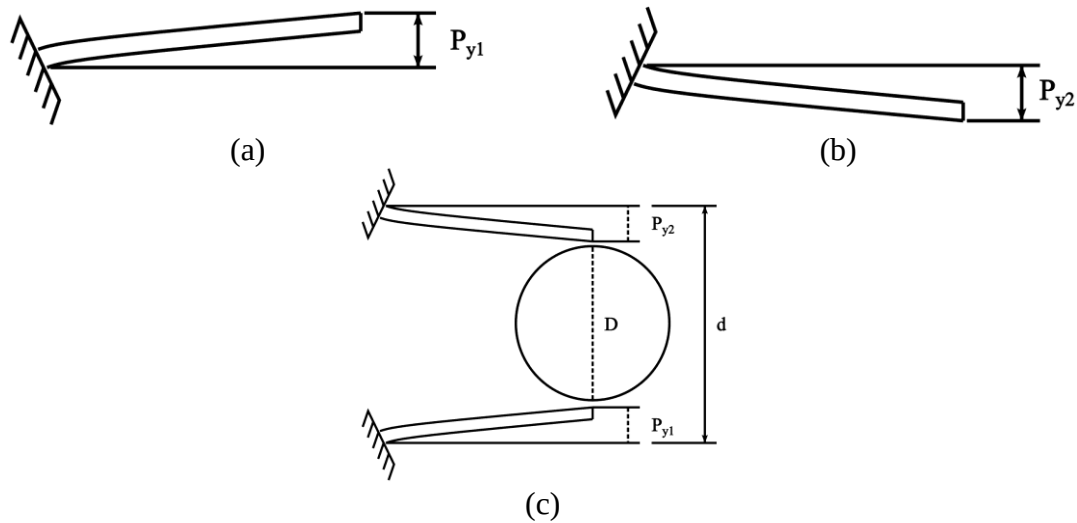


Figure 6-37.- (a) Bottom actuator, (b) top actuator and (c) gripper setup.

Another important aspect of this numerical simulation setup is that one of the beams, in this case beam-3 (top actuator), is positioned at a specific distance d relative to beam-4 (bottom actuator). This distance is calculated based on the vertical deflected position of the tip of beam-4 P_{y1} (Figure 6-37.a), beam-3 P_{y2} (Figure 6-37.b) and the diameter of the object D , as shown in Figure 6-37.c. The variables P_{y1} and P_{y2} can be calculated from the individual numerical simulation results for beam-3 and beam-4, shown in Figure 6-22 and Figure 6-24, respectively, however, instead of measuring the total displacement T_d of the tip of the beam, the vertical position P_y of the tip of the deflected beam is measured. In this case, $P_{y1} = 21.293$ mm, $P_{y2} = 18.981$ mm, and $D = 50$ mm, adding an extra millimeter to have some margin, the base of beam-3 is positioned at a distance of $d = 91.274$ mm relative to the base of beam-4.

It should be mentioned that beam-3 is positioned upside-down, this is done to ensure that both actuators direct the grasping force towards the object, as shown in Figure 5-5.

To ensure that the beams do not collide, beam-3 is initially positioned in a different plane offset by 20 mm, as shown in Figure 6-38. Later in the simulation steps, after displacing both beams, beam-3 is moved back to the same plane as beam-4.

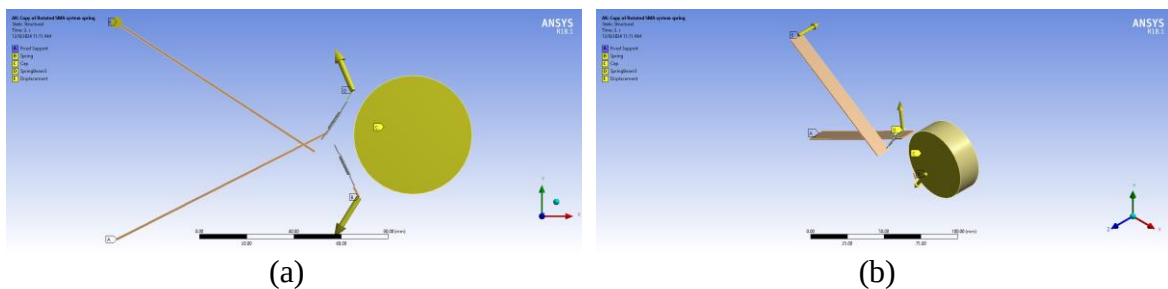


Figure 6-38.- Numerical simulation setup for case study 1, starting position (a) front view and (b) isometric view.

The numerical simulation has 4 main steps, which are described below:

1. The base of each spring is displaced by the calculated vertical and horizontal displacements using equations (6-3) to (6-7), based on the optimization algorithm results (Figure 6-39). The displacement of the spring bases induces the required input force to the tip of each beam, producing the required total displacement T_d of the tip of the beams. In this step, the actuators are still in different planes, however, since the beams are deflected, they would not collide anymore.

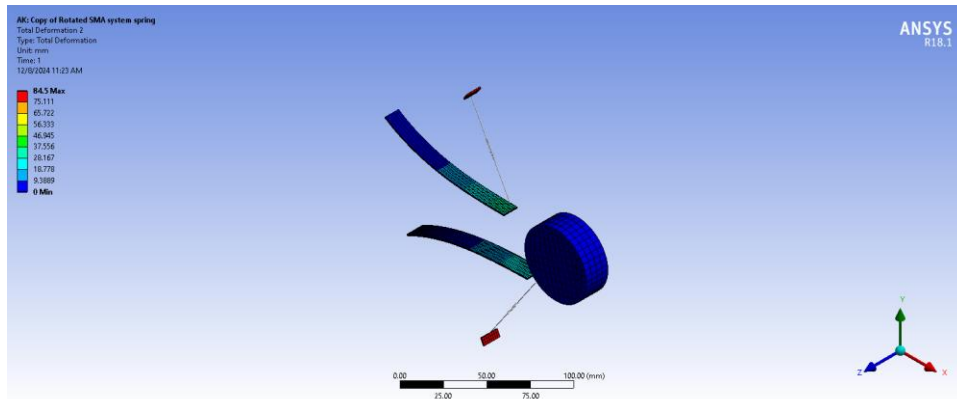


Figure 6-39.- Numerical simulation for case study 1, step 1: the gripper actuators are displaced due to the force provided by the bias springs.

2. The actuator on top (beam-3) is positioned in the same plane as the bottom actuator, ensuring that both actuators are on the same plane as the object, as shown in Figure 6-40. This step involves moving the beam and its spring base along the Z axis at the same time to ensure that the displacement of the tip of the beam stays the same.

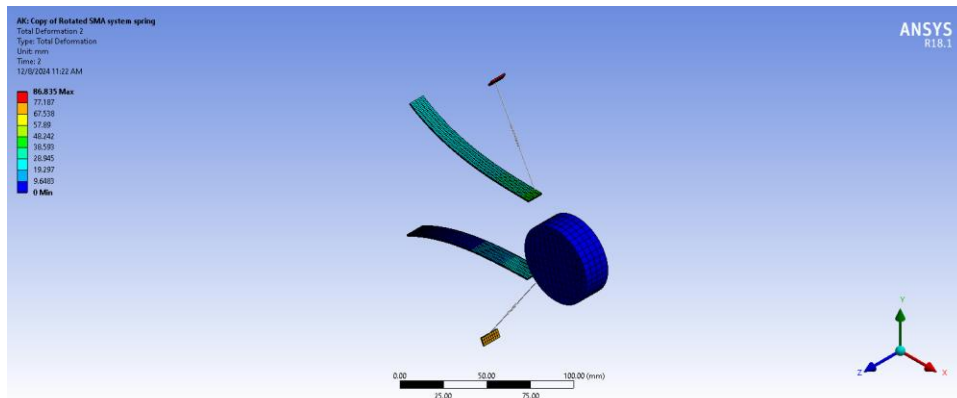


Figure 6-40.- Numerical simulation for case study 1, step 2: beam-3 is positioned the same plane as beam-4 and the object.

3. The object is positioned in place at the center of the tip of the actuators. It is important to consider that, since the actuators have slightly different mechanical properties and loads, the tips are not exactly colinear in the vertical axis, however,

the object should be positioned in the average x coordinate of both beams, in this case, beam-3 has its tip in $x = 96.15$ mm and beam-4 has its tip in $x = 97.21$ mm. The object center should be positioned at $x = 96.68$ or lower to ensure that it can be grasped by the actuators, as shown in Figure 6-41.

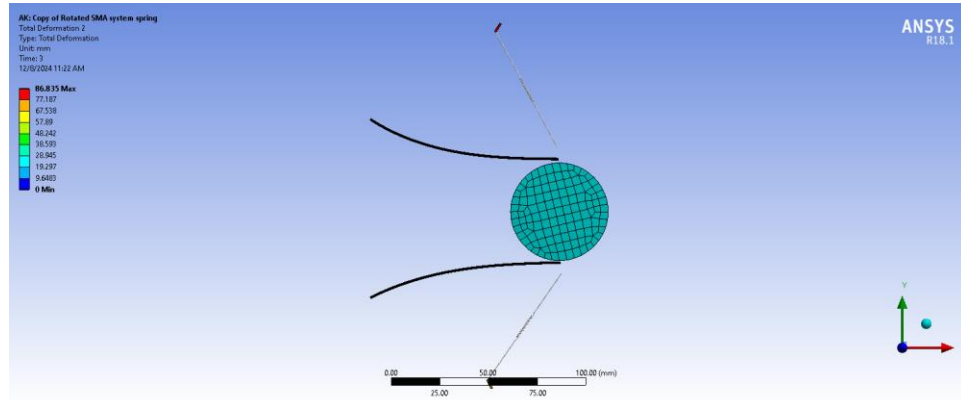


Figure 6-41.- Numerical simulation for case study 1, step 3: the object is in position to be grasped by the gripper.

4. The temperature of the actuators increases, this triggers the SME, and the actuators perform the grasping motion, as shown in Figure 6-42. In this step, the reaction force of the object was measured, totaling a reaction force of 0.6 N, ideally, this value should be 0 N to ensure that the object does not move, however, the results show that the value is low and, depending on different factors, such as the surface of the object and the gripper tips, it may be sufficiently low so that the object does fall out of the gripper tips when grasped.

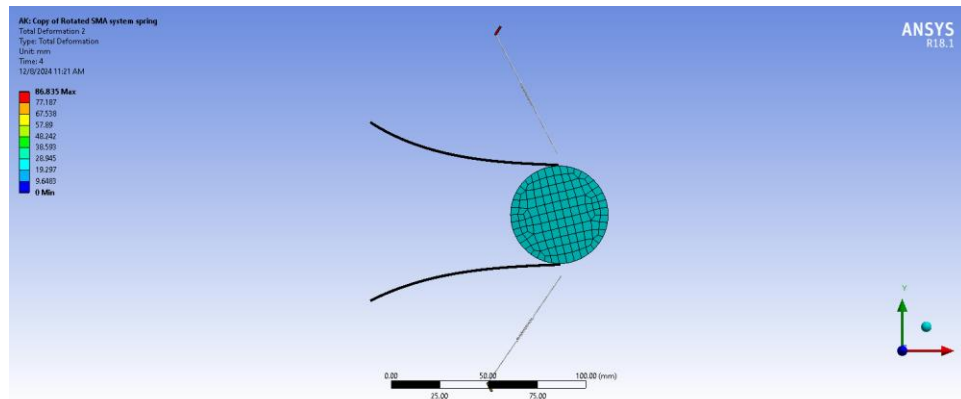


Figure 6-42.- Numerical simulation for case study 1, step 4: the object is being grasped due to the SME.

The grasping force of the gripper is given by the output force of each of the actuators, which has been analyzed separately. This gripper was designed to deliver a grasping force of 3.6 N.

This numerical simulation demonstrates the performance of the gripper design for case study 1. In reality, the gripper functionality only cycles from step 3 to step 4, since, after cooling, the bias springs return the actuators to the open position, shown in Figure 6-41. Steps 1 and 2 are only part of the building process of the gripper.

Calculating the output force of the gripper using the numerical simulation analysis is not as simple as before. Since the object is now colliding with the two actuators, the resulting force on the object is the sum of both forces, as mentioned before, the total reaction force is 0.6 N. To calculate the force that each actuator is delivering towards the object, the forces shown in Figure 6-43 should be analyzed.

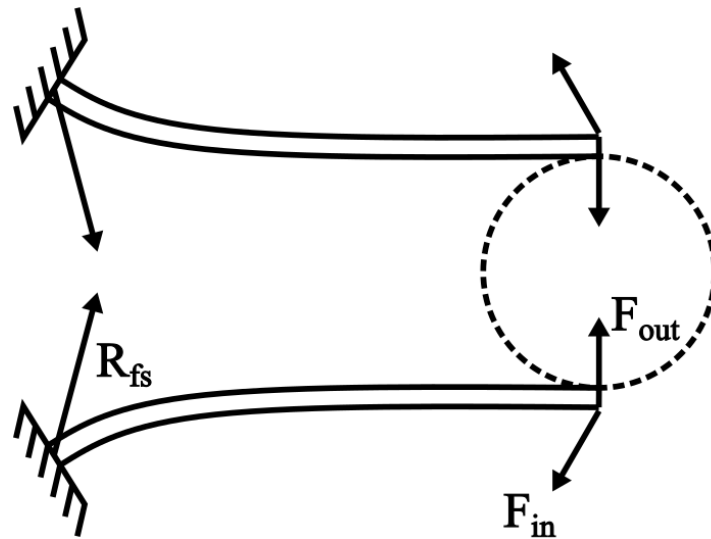


Figure 6-43.- Forces involved in the numerical simulation results analysis.

In the numerical simulation results, the input force F_{in} refers to the reaction of the spring displacement boundary condition. The fixed support reaction force R_{fs} refers to the reaction on the fixed support boundary condition. Finally, considering that the object in this numerical simulation does not move after acquiring the grasping position (Figure 6-41), the output force F_{out} delivered from each actuator can be measured by adding the components of R_{fs} and F_{in} for each actuator.

In this case, the output force F_{out} of the top actuator has a horizontal component of -0.07 N and a vertical component of -3.42 N. The bottom actuator has a horizontal component of -0.15 N and a vertical component of 3.61 N. As shown, the calculated horizontal components are relatively small compared to the vertical components, this allows the gripper to grasp the object. The average magnitude of the grasping force is 3.5 N, which results in an absolute percentage error of 2.6% compared to the objective output force of 3.6 N.

6.3.9. CASE STUDY 1 GRIPPER PROTOTYPE.

To demonstrate the functionality of this design, a prototype was built to recreate the simulation results. As a reminder, the analysis for each of the gripper actuators was done

in section 6.3.7, and this section will only showcase the functionality of the gripper prototype. Initially, the actuators are separated due to the force generated by the bias spring, and the object can fit into the open position of the gripper, as shown in Figure 6-44.

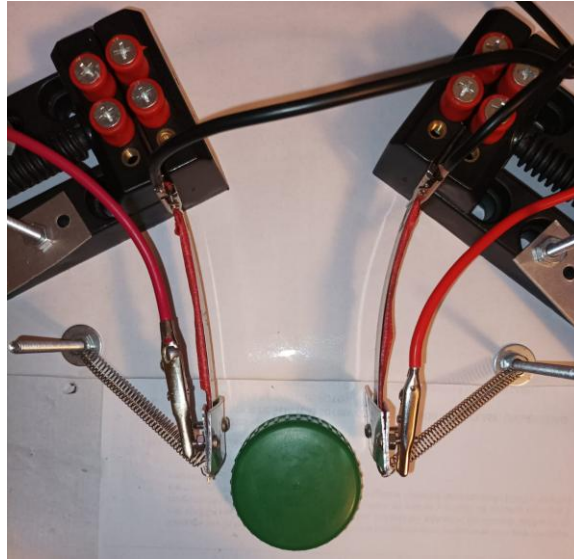


Figure 6-44.- Case study 1 gripper prototype in the martensite phase.

After increasing the temperature of the beams, using a current of 9 A running through the Ni-Cr resistance, the actuators are able to hold the object, as shown in Figure 6-45.



Figure 6-45.- Case study 1 gripper prototype in the austenite phase.

The output force of each actuator is not measured in situ using this prototype, this is because the output force of each individual actuator (beam-3 and beam-4) has already been measured in section 6.3.7.

The purpose of this prototype is to demonstrate the functionality of the gripper design, however, in order to build a proper gripper, other aspects should be considered. The most important among them are:

- The trigger mechanism for the SME: currently, the gripper actuators increase their temperature due to the Ni-Cr resistance sheet, however, the power supply delivers a current of 9 A and the time it takes for the actuators to reach the austenite finish temperature A_f is about 5 minutes 30 seconds, as shown in Figure 4-16.
- The cooling mechanism of the actuators: in the current design, the actuators passively cool down by the effect of the ambient temperature, this is not ideal since it takes about 5 minutes for the beams to be able to drop the object.
- The design of the base: the current base of the gripper consists of two clamps to hold the actuators and two bolts to connect the bias springs. This design is acceptable for prototyping since nothing is fixed in position, however, the final design of the actuators can be significantly smaller, considering that the clamps take a lot of space.

These observations are outside of the scope of this project which focuses mostly on the required calculations to ensure that the actuators are able to deliver the previously defined objective output force.

6.4. CASE STUDY 2: MAXIMUM OUTPUT FORCE.

The previous case study was based on the bibliography research of different gripper devices shown in Table 6-2. However, beam-3 and beam-4 are able to deliver a maximum output force of $F_{out} = 5.8$ N and $F_{out} = 7.8$ N respectively, as shown in Figure 6-17. Based on this information, a new case study is proposed, with the objective of creating a design that can deliver the maximum possible output force, in this case, an output force of $F_{out} = 5.8$ N, limited by beam-3. To do this, the methodology for the design of a SMA-based gripper, proposed in section 6.3 is used, however, the procedures shown in sections 6.3.1 to 6.3.3 can be skipped, since the objective output force has been redefined and the numerical model can be reused.

Based on the data shown in Figure 6-17, the required displacements for beam-3 and beam-4 to obtain an output force of $F_{out} = 5.8$ N are $T_d = 68.53$ mm and $T_d = 45.59$ mm, respectively.

6.4.1. THE TWO-BEAM MODEL.

This section cannot be skipped because the two-beam model selection is dependent on the required displacement. For beam-3, a total displacement of $T_d = 68.53$ mm is required to deliver the objective output force $F_{out} = 5.8$ N. For beam-4, the required total beam displacement is $T_d = 45.59$ mm. With this information, a similar analysis is performed as in section 6.3.4, where various simulations were run varying the two-beam model ratio between 0:100, 10:90 and 20:80. The numerical simulations also consider different input load angles φ ranging from 200 to 260 degrees. The results are compared for the same total displacement between the three two-beam models and the SMA model in Table 6-12 and Table 6-13 for beam-3 and beam-4, respectively.

T_d (mm)	φ (deg)	Force (N)				Error (%)		
		SMA	0:100	10:90	20:80	0:100	10:90	20:80
68.53	260	7.53	8.76	7.59	7.01	16.43	0.89	6.85
68.53	250	6.82	8.03	6.87	6.36	17.67	0.65	6.79
68.53	240	6.33	7.52	6.43	5.95	18.66	1.53	6.02
68.53	230	6.11	7.22	6.22	5.73	18.23	1.89	6.13
68.53	220	6.10	7.22	6.24	5.70	18.37	2.27	6.64
68.53	210	6.27	7.51	6.45	5.85	19.80	2.87	6.65
68.53	200	6.69	7.86	6.91	6.25	17.56	3.22	6.56
Average:						18.10	1.91	6.52

Table 6-12.- Beam-3 two-beam model numerical simulation results for case study 2.

T_d (mm)	φ (deg)	Force (N)				Error (%)		
		SMA	0:100	10:90	20:80	0:100	10:90	20:80
45.59	260	5.25	5.71	4.83	4.52	8.76	8.04	13.82
45.59	250	4.95	5.40	4.58	4.30	9.12	7.56	13.28
45.59	240	4.83	5.32	4.53	4.22	10.07	6.11	12.55
45.59	230	4.84	5.39	4.68	4.26	11.30	3.42	12.11
45.59	220	5.01	5.63	5.04	4.44	12.34	0.63	11.41
45.59	210	5.47	6.09	5.56	4.82	11.38	1.62	11.97
45.59	200	6.11	6.92	6.16	5.51	13.25	0.75	9.84
Average:						10.89	4.02	12.14

Table 6-13.- Beam-4 two-beam model numerical simulation results for case study 2.

As shown in Table 6-12, for beam-3, the two-beam model with a ratio of 10:90 has an average error of 1.91% when compared to the SMA model, for this reason, it should be used as the input model for the optimization algorithm.

Similarly, as shown in Table 6-13, for beam-4, the two-beam model with a ratio of 10:90 has an average error of 4.02% when compared to the SMA model. It is important to consider that the accuracy of these models is measured only for the specified total displacement. If the total displacement is modified, then the accuracy of the model should be measured again to ensure that the same model is still accurate or to decide if a different ratio should be used instead, which is why this process has to be performed again for case study 1 where, as a reminder, the two beam model with a ratio of 0:100 was preferred for beam-4.

6.4.2. OPTIMIZATION PROBLEM FORMULATION AND SOLUTION.

The optimization algorithm variable bounds for the input force F_{in} are also affected by the new output force performance objective $F_{out} = 5.8$ N. For beam-3, an input force of about 8 N is required to obtain the required beam tip displacement of $T_d = 68.53$ mm, as shown in Figure 6-17.a, using the methodology defined in section 5.3, the lower bound of the input force is set to $F_{min} = 4$ N and the upper bound of the input force is set to

$F_{max} = 12$ N. Performing the same analysis for beam-4, the required beam tip displacement of $T_d = 45.59$ mm can be achieved with an input force of about 6 N, as shown in Figure 6-17.b, then, the lower and upper bounds for the optimization algorithm are set to $F_{min} = 3$ N and $F_{max} = 9$ N, respectively. The limits for the optimization variables related to the input force angle φ and the beam rotation η are kept since the objective beam tip angle θ_{tip} is still $\theta_{tip} = 1^\circ$.

The optimization algorithm for this case study is formulated as shown below.

Minimize:

$$fun = \left| \frac{T_{di} - T_d}{T_d} \right| * 100 + \left| \frac{\theta_i - \theta_{tip}}{\theta_{tip}} \right| * 100$$

Subj. to:

$$\begin{aligned} F_{min} &\leq F_{in} \leq F_{max} \\ 202.5 &\leq \varphi \leq 270 \\ 22.5 &\leq \eta \leq 67.5 \end{aligned}$$

The optimization algorithm results for beam-3 and beam-4 are summarized in Table 6-14. For a more detailed explanation on how the algorithm works, read sections 5 and 6.3.5.

Actuator	Input force F_{in} (N)	Input force angle φ (deg)	Beam rotation η (deg)
Beam-3	6.20	268.71	61.30
Beam-4	5.36	241.78	37.24

Table 6-14.- Case study 2 optimization algorithm results for beam-3 and beam-4.

Using the results shown in Table 6-14, two numerical simulations were run for each actuator to ensure that the required displacement is reached, and the output force is obtained. The displacement results for beam-3 are shown in Figure 6-46, where the optimization algorithm results of $F_{in} = 6.2$ N, $\varphi = 268.71^\circ$ and $\eta = 61.3^\circ$ were used.

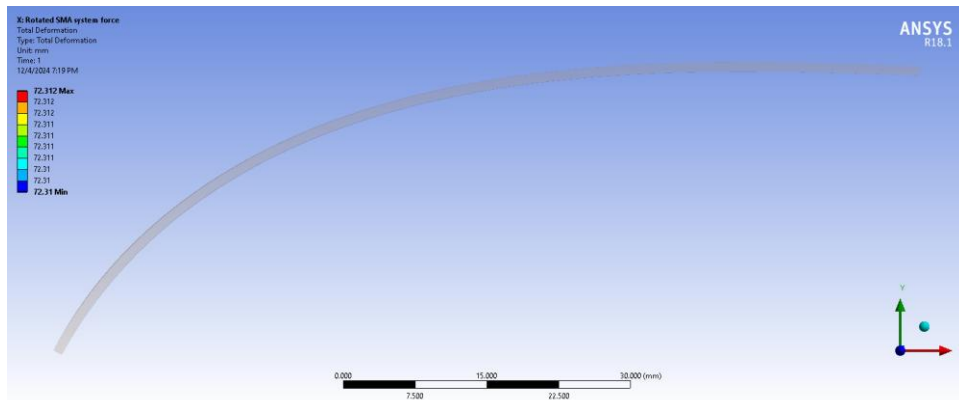


Figure 6-46.- Beam-3 tip displacement using optimization algorithm results for case study 2.

In this case, the maximum equivalent elastic strain is calculated as 0.7%, this value is below the maximum recoverable strain of 8%, so it should not cause an unrecoverable strain on the material.

The numerical simulation tip displacement is $T_d = 72.31$ mm. The absolute percentage error compared to the required displacement of 68.53 mm is calculated as 5.51% using equation (6-1). The output force is calculated in the numerical simulation by positioning an obstruction on the tip of the beam, at a 2 mm distance from the tip, then increasing the temperature of the actuator to trigger the SME. The output force is calculated as $F_{out} = 6.09$ N. This results in an absolute percentage error of 5%. Furthermore, the rotation of the beam $\eta = 61.3^\circ$ allows the tip of the beam to be almost horizontal, allowing the grasping force to be applied properly to an object, as shown in Figure 6-47. The results are satisfactory based on the calculated errors.

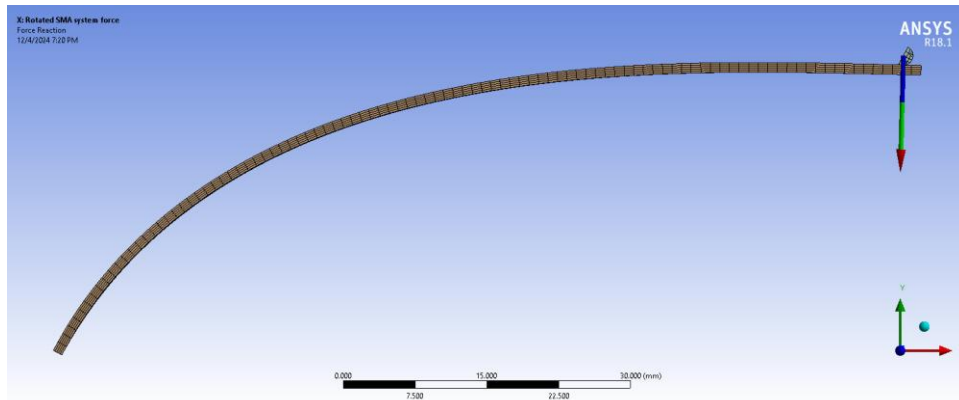


Figure 6-47.- Beam-3 output force reaction direction for case study 2.

The required displacement for beam-4 is 45.59 mm. Using the optimization algorithm results of $F_{in} = 5.36$ N with an input force angle of $\varphi = 241.78^\circ$ and a beam rotation of $\eta = 37.24^\circ$, the obtained beam tip displacement is $T_d = 43.37$ mm, as shown in Figure 6-48. The calculated absolute percentage error for this result is 4.86% when compared to the required beam tip displacement of 45.59 mm.

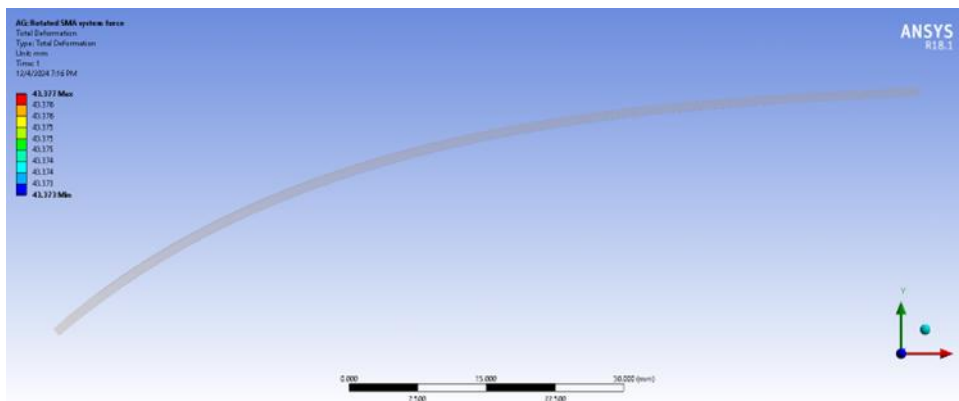


Figure 6-48.- Beam-4 tip displacement using optimization algorithm results for case study 2.

The maximum equivalent elastic strain is calculated as 0.5% using the numerical simulation, and it should not cause any unrecoverable strains.

A numerical simulation to calculate the output force for beam-4 is also performed. The output force is calculated as $F_{out} = 5.6$ N. This results in an absolute percentage error of 3.44%, compared to the objective output force of 5.8 N.

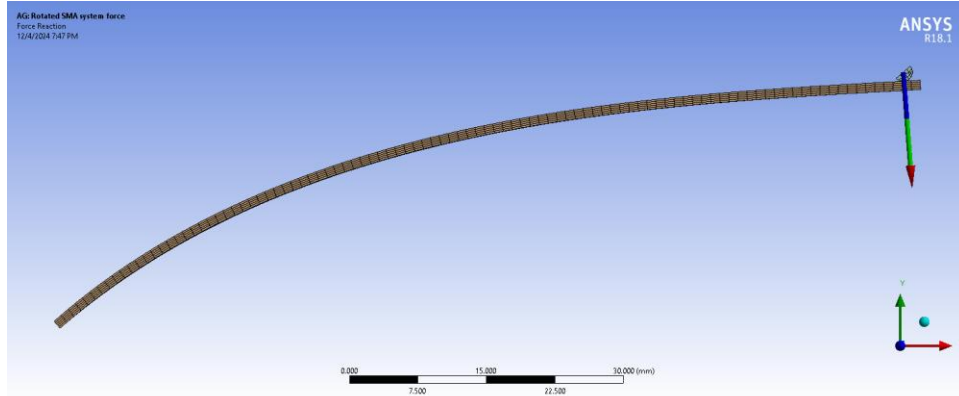


Figure 6-49.- Beam-4 output force reaction direction for case study 2.

As shown in Figure 6-49, the output force is delivered properly towards the obstruction, due to the beam rotation $\eta = 37.24^\circ$ calculated by the optimization algorithm. The calculated error results are also satisfactory for beam-4.

6.4.3. EXPERIMENTAL RESULTS.

To reproduce the results in an experimental setup, the input force is replaced by a bias spring with a spring constant of $k_s = 0.087$ with an initial length of $L_{s1} = 19.2$ mm. The same experimental setup is used as in section 6.3.7. For beam-3 the bias spring needs to travel a distance of $x = 71.26$ mm to deliver the required input force of $F_{in} = 6.2$ N, as calculated using equation (6-2), the final length of the spring is calculated as $L_{s2} = 90.46$ mm using equation (6-3). Similarly to the experimental results shown in section 6.3.7, the beam rotation $\eta = 61.3^\circ$ is subtracted from the required input force angle $\varphi = 268.71^\circ$, to be able to perform the analysis without rotating the beam, based on this, the spring is positioned at an angle of 207.41 degrees, to account for the rotation of the beam.

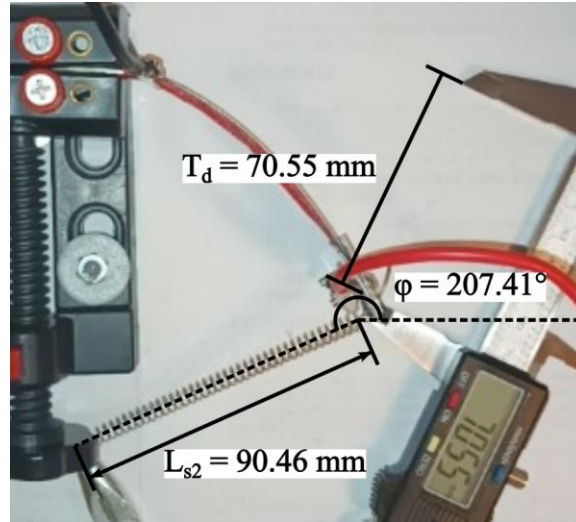


Figure 6-50.- Beam-3 experimental test displacement using a bias spring for case study 2.

As shown in Figure 6-50, the spring travels a distance of $T_d = 70.55$ mm, this results in an error of 2.94%, compared to the required displacement of 68.53 mm.

After increasing the temperature of the beam by connecting the power supply, the output force increases to $F_{out} = 7.1$ N, as shown in Figure 6-51. The output force has an error of 22.41% when compared to the objective output force of 5.8 N.

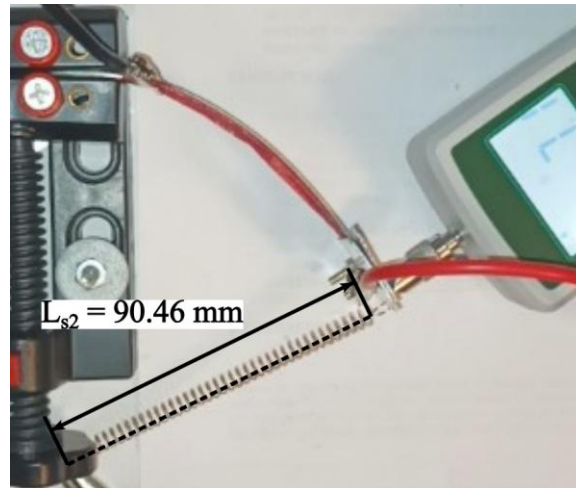


Figure 6-51.- Beam-3 experimental test output force using a bias spring for case study 2.

The same analysis is performed using beam-4, using the optimization algorithm results of the required input force $F_{in} = 5.36$ N, the required input force angle $\phi = 241.78^\circ$ and the required beam rotation $\eta = 37.24^\circ$. In this case, the bias spring has to deliver an input force of 5.36 N, using equation (6-2), the distance that the spring needs to travel is calculated as $x = 61.6$ mm, then, the final length of the spring can be calculated using equation (6-3) as $L_{s2} = 80.8$ mm. The rotation of the beam $\eta = 37.24^\circ$ is subtracted

from the input force angle $\varphi = 241.78^\circ$, this results in the spring being positioned at an angle of 204.54 degrees. Using this experimental setup, the beam displacement results in $T_d = 50.30$ mm, with a required displacement of 45.5 mm, as shown in Figure 6-52, resulting in an absolute percentage error of 10.54%.

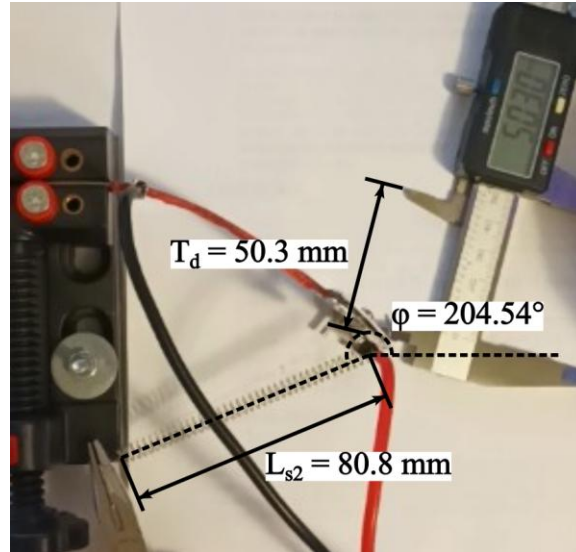


Figure 6-52.- Beam-4 experimental test displacement using a bias spring for case study 2.

After turning on the power supply, the beam delivers an output force of $F_{out} = 6.4$ N, as shown in Figure 6-53, resulting in an absolute percentage error of 10.34%, compared to the objective output force of 5.8 N.

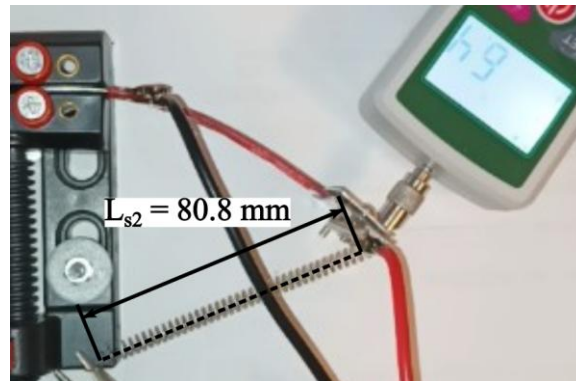


Figure 6-53.- Beam-4 experimental test output force using a bias spring for case study 2.

The results for the maximum output force design are less accurate than the ones shown in section 6.3.7. The actuator beam-3 can deliver a maximum output force of $F_{out} = 7.1$ N, this exceeds the objective output force of $F_{out} = 5.8$ N, resulting in an absolute percentage error of 22.41%, which is inconsistent with the numerical simulation results, where the output force was calculated as $F_{out} = 6.09$ N. Similarly, for beam-4, the experimental results show that, using the optimization algorithm results, the actuator can

deliver a maximum output force of $F_{out} = 6.4$ N. This value exceeds the output force objective with an absolute percentage error of 10.34%, this is also inconsistent with the numerical simulation results, where the output force was calculated as $F_{out} = 5.6$ N. In both cases, the numerical simulation results for F are closer to the objective output force of $F_{out} = 5.8$ N. This means the models are probably imprecise when higher displacements are involved, since the numerical simulations using the optimization algorithm results show that the output forces should have been closer to the specified objective.

6.4.4. CASE STUDY 2 GRIPPER DESIGN NUMERICAL SIMULATION.

Similar to case study 1, a numerical simulation is set up in order to demonstrate the functionality of the gripper design, based on the individual actuators numerical simulation results. The numerical simulation consists of the same steps shown in section 6.3.8, now utilizing the required spring base displacements and beam rotations for case study 2. For this reason, the actuators are initially at a different angle compared to Figure 6-38, as shown in Figure 6-54.

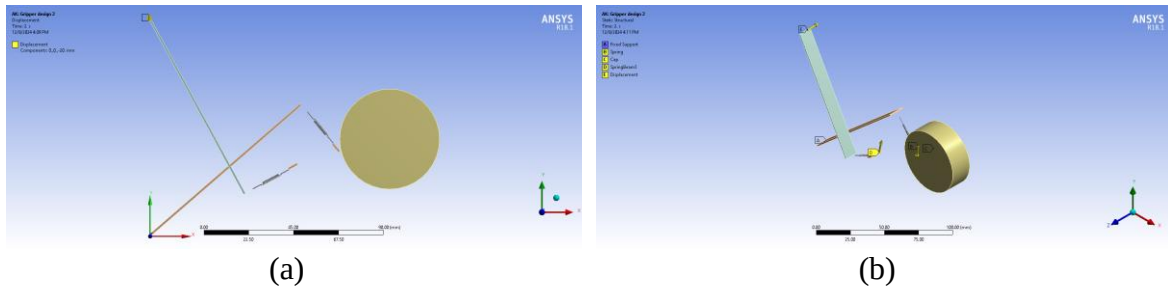


Figure 6-54.- Numerical simulation setup for case study 2, starting position (a) front view and (b) isometric view.

The steps for this numerical simulation are summarized below:

1. The actuators are displaced by the bias springs due to the movement of the spring bases (Figure 6-55.a).
2. Beam-3 (top) is translated to the same plane as beam-4 (bottom) to be able to grasp the object (Figure 6-55.b).
3. The object moves into grasping position in between the tips of the actuators (Figure 6-55.c).
4. The temperature of the actuators increases, and the SME is triggered (Figure 6-55.d).

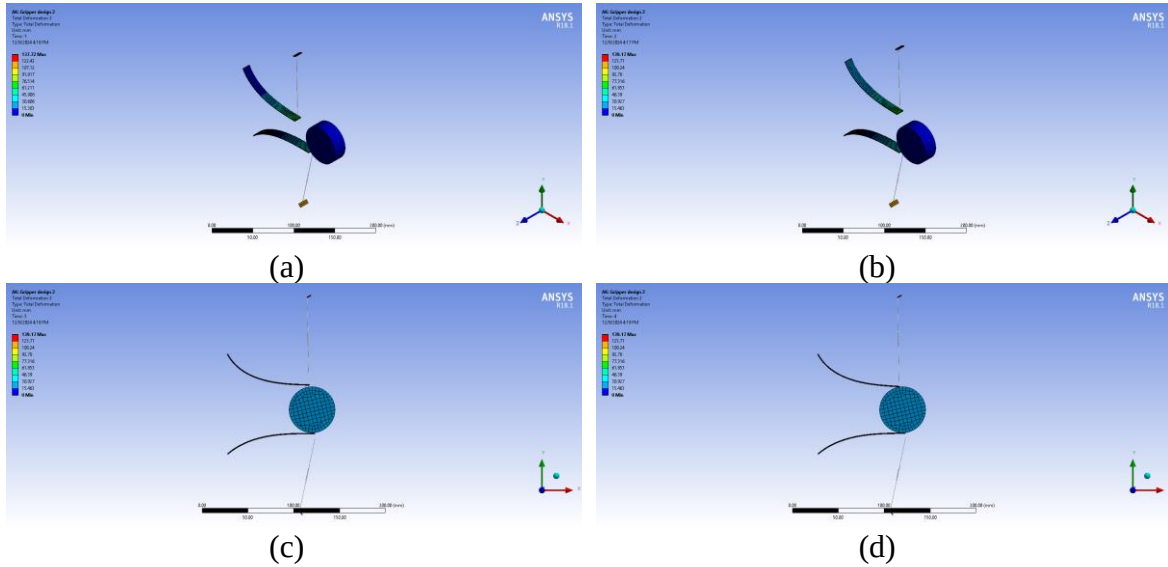


Figure 6-55.- Numerical simulation results steps: (a) Step 1: beam displacement, (b) Step 2: beam translation in plane, (c) Step 3: object position to be grasped. (d) Step 4: shape memory effect trigger.

In this gripper design, the reaction force on the center of the object is equal to 0.86 N, as mentioned in section 6.3.8, ideally this value should be equal to 0, however, its effect also depends on the friction of the surface of the actuators and the object.

The grasping force can be calculated by following the same procedure used in section 6.3.8. Where the reaction force of the fixed support R_{fs} and the input force F_{in} applied by the springs can be added to calculate the output force F_{out} of each actuator. In this case, the top actuator output force has an horizontal component of -0.13 N and a vertical component of -5.73 N, while the bottom actuator has an horizontal component of -0.04 N and a vertical component of 5.96 N. As expected, the horizontal components are relatively small when compared to the vertical components, and the average magnitude of the output force is 5.84 N, which results in an absolute percentage error of 0.6% when compared to the objective output force of 5.8 N.

The prototype built in section 6.3.9 serves as a proof of concept of the design prototype and it is not necessary in this section since each actuator has been analyzed separately.

As stated before, the scope of this thesis project is mainly focused on the calculations required to implement the SMA beams as gripper actuators, for this reason, the design of the actuator tips, along with the design of a faster SME triggering method, a better cooling solution and the design of a smaller base for the parts of the gripper shall be left as future work.

7. CONCLUSIONS.

In this thesis project, a methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper was developed. This project involves the implementation and adaptation of the pseudo-rigid body model with three revolute joints, which serves as a tool for the calculation of large deflection of beams connected in series. The PRBM was implemented in MATLAB allowing it to be coupled with an optimization algorithm in order to minimize the displacement error of a SMA cantilever beam in order to obtain a previously specified output force. However, the optimization algorithms cannot serve their purpose unless the material properties of the SMAs are known. In this thesis project, cantilever beam displacement theories were used in addition to numerical simulation software in order to estimate the mechanical properties of four Ni-Ti shape memory alloys. Two case studies were analyzed making use of the proposed methodology of this project, in general, the results show that the estimated mechanical properties of the SMA materials were accurate for lower displacement ranges, while some more work is required to improve the results when higher displacements are involved.

7.1. MATLAB IMPLEMENTATION OF THE PRBM AND OPTIMIZATION ALGORITHMS.

Using MATLAB to implement the pseudo-rigid body model static equations allowed the incorporation of optimization algorithms to calculate the required loading conditions of the actuators in order to exploit the SME to generate an output force that can serve as the output force of a gripper. The experimental results demonstrate that the optimization algorithm did accurately calculate the required loading conditions to deliver the specified output force. The limitation of the PRBM of not being able to consider the plastic region of the material was overcome by utilizing the two-beam model.

7.2. ESTIMATION OF MECHANICAL PROPERTIES OF CANTILEVER BEAMS USING NUMERICAL SIMULATIONS AND EXPERIMENTAL DATA.

Various experimental tests were performed in order to be able to estimate the mechanical properties of four shape memory alloy actuators by comparing the force-displacement curve of the experimental data with numerical simulation results. To do this, it is important to understand the effect of the mechanical properties of the material on the behavior of the force-displacement curve. For SMA cantilever beams, there are two main phases one should focus on, in the martensite phase, the martensite phase elastic modulus and the elastic limit have the most impact on the behavior of the force-displacement curve. For the austenite phase, the austenite phase elastic modulus, the elastic limit and the temperature scaling parameter have an important effect on the force-displacement curve. For both phases, the respective elastic modulus alters the shape of the entire force-displacement curve, while the elastic limit in both cases initially starts by not having an effect, but at one point the force-displacement curve diverges depending on the elastic limit. The point in question is the load where the elastic limit has been reached and the cross section of the beam starts to transform from an elastic regime to a plastic regime. It is also important to consider that the tangent modulus of the material also has an

important effect on the force-displacement curve, altering the slope of the curve after the elastic limit has been reached.

The estimated mechanical properties allowed the optimization algorithm to calculate the required conditions to reach a specific total displacement of the actuators. The maximum error in this regard was 10.54% in case study 2 for beam-4, with this in mind, it can be concluded that the estimation approach used in this thesis project is reliable, and the results could potentially improve if the analysis is performed with more experimental data points in the force-displacement curve of the cantilever beam test, and by performing the test with different force angles to improve accuracy for different loading conditions.

In this work the weight of the beam is not considered under the assumption made in section 4.3, where it was stated that the weight of the beam only contributes to 3.69% of the total displacement of the beam with a load of 100 g. It was also mentioned that the contribution towards the displacement of the beam decreases as the load increases since the weight of the beam remains the same. This could be related to the inaccuracies of the model when the load is 100g, however, it is unlikely that this had a significant impact on the results of the case studies since the required displacements were not close to the displacement range (4 – 6 mm) where the error increased drastically for the model compared to the experimental results (see Table 6-5 and Table 6-7).

An important point that should be considered regarding the experimental analysis of the martensite and austenite displacement of the beams is that in this project the numerical simulation models are based on an experimental test. It is currently unknown if performing multiple tests would lead to better results, at least for case study 1. As reference, in case study 1 the displacement error results in 7.17% for beam-3 and the displacement error results in 0.6% for beam-4. In both cases, the output force error is 8.33%. Creating a numerical simulation model from the average displacement of multiple experimental tests may help to reduce these error values, however, this was not done since the material may suffer alterations on its thermal properties caused by the thermal cycling, specifically, the transformation temperatures may be altered and the results from different experimental tests may vary, this is a known behavior of Ni-Ti SMAs [57], [58]. The effects of thermal cycling on the materials should be considered but would require a more in-depth analysis and is outside the scope of this project.

7.3. NUMERICAL SIMULATION MODELING.

As mentioned before, several numerical simulations were used to perform different types of analyses, such as the sensitivity analyses. The numerical simulation models can be used to estimate the maximum output force of a SMA actuator. They can also help to verify that the optimization algorithm results are accurate, by modeling the displacement of the rotated beam with the solution input load.

Numerical simulation models were also used to select the best two-beam model ratio for the estimation of the displacement of a SMA beam, based on the required displacement to attain the objective output force.

The results for case study 2 demonstrate that it is important to test the model for various loading conditions, since a maximum objective output force absolute percentage error of 22.41% was obtained, potentially due to inaccuracies of the model at high displacement ranges or with the specific loading condition presented in that case. This is easier to understand based on the following: the numerical simulation model for beam-3 was defined based on experimental results where the displacements ranged from 14.44 mm to 59.29 mm in the martensite phase, and 4.16 mm to 30.46 mm in the austenite phase, as shown in Figure 7-1.

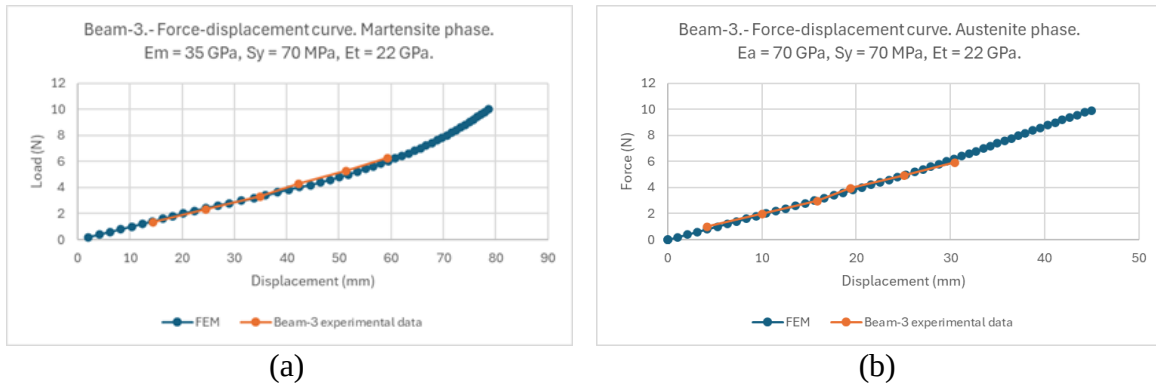


Figure 7-1.- Numerical simulation model force-displacement curve for beam-3 in (a) martensite phase and (b) austenite phase.

As a reminder, the required displacement to deliver an output force of 5.8 N with beam-3 was calculated as 68.53 mm, this value is slightly above the maximum displacement obtained in the martensite phase experimental analysis, in this case, when displacing the actuator in the martensite phase, the obtained error was 2.94%. However, the required displacement of 68.53 mm is much larger than the maximum displacement obtained in the austenite phase, the model was not verified for such a displacement range, and as a consequence, the results were inaccurate. It is important to verify the numerical simulation model for a broad range of displacements to ensure that it represents the material with as much accuracy as possible.

7.4. CONTRIBUTIONS.

- In this project a methodology for the implementation of shape memory alloy sheets as the activation method of a compliant gripper was proposed and tested.
- A PRBM for cantilever beams in series was also adapted for the purpose of this project, and its mathematical equations were implemented in MATLAB to be used as the input model of an optimization algorithm.
- The use of FEM numerical simulation analysis as a tool for the design of a SMA-based compliant gripper was also introduced, involving the estimation of mechanical properties of materials based on experimental data, the calculation of the output force of a SMA-based actuator, the process of performing a sensitivity analysis and the simulation of the functionality of a SMA-based gripper.
- A prototype of a shape memory alloy-based compliant gripper was built, to demonstrate the results of the proposed methodology.

8. FUTURE WORK.

As future work, the gripper design should be improved in the following areas:

- The response time of the gripper could be improved with a better SME triggering method that increases the temperature of the actuator faster. One possible solution is to implement a fast discharge device that is capable of delivering a high current in a small amount of time to heat up the Ni-Cr resistance.
- The cooling of the Ni-Ti alloys is also very slow, and it currently cannot compete with grippers actuated by DC motors. A better passive or active cooling solution should be analyzed and implemented to improve the deactivation time of the gripper jaws.
- A deeper analysis of the behavior of the beams for relatively high displacements (>60 mm) should be performed, since, as shown in case study 2, the output force absolute percentage error in this displacement range was severely higher than in case study 1, where the error was more acceptable.

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